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Title: Approaches to Formalisation of the Logic of Classes and its Extension to ZFC

Abstract: The logic of classes, under the name the virtual theory of classes (VTC), was introduced by Quine as a flexible neutral tool for comparison of different approaches to set theory. It specifies in a non-committal way the conditions for reasoning with set theoretic constructs. Quine developed VTC on the basis of classical logic, but it was also dealt with by Dana Scott in the version founded on positive free logic. An independent variant of the Fregean logic of classes was proposed by Neil Tennant on the basis of negative free logic. Despite the differences, all these approaches can be treated as forms of weak logicism reconstructing the logic of our discourse on sets but without assuming the existence of some special sets. We can use them as a neutral platform for the development and comparison of alternative theories of sets, like ZF, NGB or NF.

Neither VTC nor Scott's theory has so far been explored proof-theoretically although Tennant provided a natural deduction system for his Fregean logic of classes. We present sequent calculi for all variants. In particular, calculi for Quine's and Scott's logics satisfy cut elimination, generalised subformula property and are provably consistent, also in the intuitionistic variant. Sequent calculus for Tennant's logic also satisfies cut elimination but is less satisfactory. For the sake of illustration we show how these calculi can be extended to cover ZFC by means of (systems of) rules. These sequent formalisations of ZFC also preserve cut elimination. Moreover, some of its subsystems in the free version are also provably consistent, in contrast to their classical counterparts.