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# ON A MINIMAL NON-ALETHIC LOGIC

**1.** Non-alethic logic was introduced in da Costa [1]. In this kind of logic the principles of tertium non datur and of contradiction are not valid; furthermore, non-alethic logic constitutes a generalization of both paraconsistent and paracomplete logics (cf. da Costa and Marconi [2] and [3]).

In this note we present a minimal non-alethic logic $A$, which can be employed as a basis for a deontic logic that does not exclude *ab inintio morral dilemmas* as real deadlocks, and considers them only *as prima facie* difficulties. The deontic logic founded on $A$ is also compatible with the existence of deontic gaps, i.e., deontic situations whose proper deontic status can not be directly settled.

Consequently, $A$ may be the starting point of systems of deontic logic apt to cope with ethical and juridical issues of fundamental nature.

Our terminology and symbolism are those of Kleene [5], with obvious adaptations and extensions. We limit ourselves to the level of the propositional calculus, but our main results can be extended to the first-order level.

**2.** The primitive symbols of $A$ are the following:

(a) Propositional variables (an infinitely denumerable set of variables);
(b) The connectives: $\supset$ (implication), & (conjunction), $\wedge$ (disjunction), and $\neg$ (negation); equivalence, $\equiv$, is defined as usual;
(c) Parentheses.

We define formula, eliminate parentheses, etc., as in Kleene [5]. Capital Latin letters will always stand for formulas, and capital Greek letters for sets of formulas.

The postulates (axiom schemes and primitive rule of inference) are the following, where $A^\circ$ is an abbreviation for $\neg(A\&\neg A)$, and $A^*$ is an abbreviation for $A \vee \neg A$:

1. $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$
2. $A \supset (B \supset A)$
3. $A, A \supset B/B$
4. $A \supset (B \supset A\&B)$
5. $A\&B \supset A$
6. $A\&B \supset B$
7. $A \supset A \vee B$
8. $B \supset A \vee B$
9. $(A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$
10. $((A \supset B) \supset A) \supset A$
11. $A^*\&B^\circ \supset ((A \supset B) \supset ((A \supset \neg B) \supset \neg A))$
12. $A^* \supset (\neg\neg A \supset A)$
13. $A^\circ \supset (A \supset \neg\neg A)\&(A \supset (\neg A \supset B))$
14. $A^\circ\&B^\circ \supset ((A \supset B)^\circ\&(A\&B)^\circ\&(A \vee B)^\circ\&(\neg A)^\circ)$
15. $A^*\&B^* \supset ((A \supset B)^*\&(A\&B)^*\&(A \vee B)^*\&(\neg A)^*)$

Definition. $\sim A \stackrel{\mathrm{df}}{=} \neg A\&A^\circ\&A^*$

When the propositional variables of a set of formulas satisfy the principle of tertium non datur, $A \vee \neg A$, for this set of formulas is valid the calculus $C_1$ (cf. da Costa and Marconi [3]), which is paraconsistent. On the other hand, if the propositional variables of a set of formulas satisfy the principle of contradiction, $\neg(A\&A)$, then for this set of formulas is essentially valid the calculus $P_1$ (da Costa and Marconi [2]).

$A$ possesses several interesting properties, for instance, the following:

1) In $A$ all rules and valid schemes of classical positive logic are true (we have, for example: if $\Gamma, A \vdash B$, then $\Gamma \vdash A \supset B$; if $\Gamma, A \vdash C$ and $\Gamma, B \vdash C$, then $\Gamma, A \vee B \vdash C$; $A, B \vdash A\&B$);

2) $A$ is not decidable by finite matrices;

3) If $\Gamma \vdash A$ is true in the classical propositional calculus, $\Gamma^\circ = \{\gamma^\circ : \gamma$ is a propositional variable occurring in a formula of $\Gamma\}$ and $\Gamma^* = \{\gamma^* : \gamma$ is a propositional variable occurring in a formula of $\Gamma\}$, then $\Gamma, \Gamma^\circ, \Gamma^* \vdash A$ in $A$;

4) In $A$ the strong negation $\sim$ has all properties of classical negation;

5) In $A$ the schemes $A \supset (\neg A \supset B), \neg A \supset (A \supset B)$, $A \& \neg A \supset B$, $(A \equiv \neg A) \supset B$, $A \vee \neg A$ and $\neg(A \& \neg A)$ are not valid;

6) $A$ has a nice semantics of valuations.

So $A$ is both a paraconsistent and a paracomplete logic.

**3.** We now introduce a propositional deontic logic, $DA$, having $A$ as its basis: the primitive symbols of $DA$ are those of $A$, plus the obligation operator $O$. The postulates of $DA$ are 1-15 above, to which we adjoin the following standard deontic principles:

| | |
|---|---|
| I | $O(A \supset B) \supset (OA \supset OB)$ |
| II | $OA \supset \sim O \ \sim A$ |
| III | $A^\circ \supset (OA)^\circ$ |
| IV | $A^* \supset (OA)^*$ |
| V | $A/OA$ |

Definition.

$FA \stackrel{\text{df}}{=} O\neg A \qquad PA \stackrel{\text{df}}{=} \neg\ O\neg A$

$\tilde{F}A \stackrel{\text{df}}{=} O \sim A \qquad \tilde{P}A \stackrel{\text{df}}{=} \sim O \sim A$

Theorem 1. *In $DA$, we have:*

| | |
|---|---|
| $\vdash (OA \& O \sim A) \supset OB$ | $\vdash \sim (OA \& \sim OA)$ |
| $\vdash OA \vee \sim OA$ | $\vdash O \sim A \supset \sim OA$ |
| $\vdash OA \supset O(A \vee B)$ | $\vdash OB \supset O(A \vee B)$ |
| $\vdash OA \& O(A \supset B) \supset OB$ | |

Theorem 2. *In $DA$ are not derivable the following schemes:*

| | |
|---|---|
| $O(A \& \neg A) \supset OB$ | $O(A \& \neg A) \supset O\neg B$ |
| $(OA \& O\neg A) \supset OB$ | $O\neg(A \& \neg A)$ |
| $O(A \vee \neg A)$ | $O(A \equiv \neg A) \supset OB$ |
| $OA \supset O(\neg A \supset B)$ | $O\neg A \& (A \vee B) \supset OB$ |
| $(FA \& F\neg A) \supset OB)$ | $(FA \& F\neg A) \supset FB$ |

We easily develop a Kripke semantics for $DA$, with the help of the semantics of valuations of $A$.

**4.** Since $A$ is paracomplete, the logic $DA$ can be used to tread, in a smooth way, the common problems originated by the gaps in the ethical and juridical systems.

On the other hand, taking onto account the praconsistent character of $A$ and $DA$, this second logic can cope with the problems of ethical dilemmas and of juridical contradictions.

## References

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