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PEIRCED CLEAN THROUGH

What is the effect of adding Peirce's Law to $R_{\rightarrow}$? Answer, disaster. For

1. $p \rightarrow q \rightarrow p \rightarrow p$ Peirce
2. $(p \rightarrow .q \rightarrow p) \rightarrow p \rightarrow p$ 1; Substitution of $q \rightarrow p$ for q
3. $(p \circ q \rightarrow p) \rightarrow p \rightarrow p$ 2; Importation Replacement of Equivalents
4. But, in $R_{\rightarrow}$, $(A \rightarrow p \rightarrow p \rightarrow p \rightarrow .A \rightarrow p$
5. So, $p \circ q \rightarrow p$ 3, 4; $\rightarrow E$
6. I.e. $p \rightarrow .q \rightarrow p$ 5; Exportation

But, on Tarski's axiomatization of classical implicational system $TV_{\rightarrow}$, 1 and 6 together with the suffixing axiom

7. $p \rightarrow q \rightarrow .q \rightarrow r \rightarrow .p \rightarrow r$

of $R_{\rightarrow}$ yield classical tautologies in $\rightarrow$. All of them. So $R_{\rightarrow}$ with Peirce is peirced clean through.

Strictly speaking, the above proof is invalid in that fusion $\circ$ is of course no connective of $R_{\rightarrow}$. But this, in fact, as a minor point, and the whole proof can be written out in longhand if desired.

Further, what is the effect of adding weaker versions of Peirce's Law to $R_{\rightarrow}$? It is of interest (though not conclusive, since the effect obviously depends on the direction of weakening) to note that the direction of weakening which produces the Łukasiewicz many valued logics doesn't weaken anything on $R_{\rightarrow}$ insights.

Rose [1] showed that the characteristic axiom of Łukasiewicz $n+1$ valued logic is, essentially,

1° $p^n \to q \to p \to p$,

where $p^1 = p, p^2 = p \circ p$, etc.

By the same argument as above, we get from 1° to

5° $p^n \circ q \to p$

But in $R_{\to}$, we can contract on 5° to get 5 and 6 directly. (In some sense, this direction of collapse is not surprising, since, from the Łukasiewicz viewpoint, contraction and Peirce are two sides of the same coin.) So not only Peirce itself but all its Łukasiewicz-style weakening also collapse $R_{\to}$ to $TV_{\to}$.

References

[1] A. Rose, *Formalizations du Calculus Propositionnel Iplicatif à m-valuers de Lukasiewicz*, **Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences**, Série A, 243 (1956), pp. 1263–1264.

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