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A QUESTION ABOUT JOIN-SEMILATTICES

Let $(L, \vee, 1)$ be a non-degenerate join-semilattice with unit, and let $\mathcal{R}$ be the totality of its maximal ideals. For any $A \subseteq L$ we set $r(A) = \{R \in \mathcal{R} : A \cap R \neq \emptyset\}, A^{\perp} = \{y \in L : x \vee y = 1$ for every $x \in A\}$, and $V(A) = (A^{\perp})^{\perp}$. Clearly, V is a closure. (For motivation and details cf. our paper "A topology for logical space" this **Bulletin**, vol. 13 (1984), no. 4)

Next assume that L satisfies the following "compactness" condition: for any $A \subset L$, and any $R \in \mathcal{R}$,

$$\bigwedge_{A_i \in FinA} A_i^{\perp} \cap R \neq \emptyset \Rightarrow A^{\perp} \cap R \neq \emptyset,$$

where $FinA$ are all the finite subsets of A, including the empty one. We have then: $r(A) = r(V(A))$.

Assume, moreover, that L satisfies the Descending Chain Condition. We have then: $r(A) = r(MinA)$, where $MinA$ are all the minimal elements of A. Clearly, if $A \neq \emptyset$, $MinA$ is an antichain.

Finally consider the antichain $A' = MinV(A)$. It may contain a proper subchain B such that $r(B) = r(A')$. Indeed, suppose $A' = \{x, y\}$, and $x \in R \Rightarrow y \in R$, for all $R \in \mathcal{R}$. Then $r(\{x, y\}) = r(\{y\})$. (Actually we have in general: if $B \subset A$ and $r(B) = r(A)$, then $r(A - B) = r(B)$.)

The question is: under what properties of L – short of all its maximal ideals being finite – does an antichain $MinV(A)$ always contain a *minimal* subchain B such that $r(B) = r(MinV(A))$?

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