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ON LOGICS OF DEEDS

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I. Language of the logics of deeds

The following symbols constitute the alphabet of the n-th logic of deeds LDn: p_i (i is an arbitrary natural number greater than zero), $-$, T, $d_i \dots d_n$, $\vee$ (disjunction), $\wedge$ (conjunction), $\sim$ (negation), (,). Expressions of the form p_i might be viewed as sentential variables, individual variables, sentential constants or, at last, as individual variables, sentential constants or, at last, as individual constants. (This freedom of interpretation of expressions of the form p_i may be traced back to the fact that there are no substitution rules in LDn systems, and there are neither any quantifiers appearing in the languages of these systems). It is perhaps worth while to observe that it is of no essential importance for the subscripts accompanying the letter p to be natural numbers. If the set of all subscripts accompanying the letter p was uncountable the subsequent constructions of the logics of deeds LDn as well as their semantics would undergo no serious changes.

The set of all terms is the smallest set X that has the following two properties: 1. all p_i are in X, 2. if α is in X, then $-\alpha$ is also in X.

The set of all T-formulae is the smallest set comprising all the expressions of the form $\alpha T \beta$, where α and β are terms, and closed under the operations of joining the expressions by means of the disjunction and conjunction connectives, preceding them with the negation connective and putting the expressions in brackets. The atomic T-formulae have the form

$\alpha T\beta$, whereas the elementary T-formulae have the form p_iTp_i, $p_it - p_i$, $-p_iTp_i$, $-p_iT - p_i$. The latter formulae are abbreviated, respectively, as p_i^0, p_i^2, p_i^3, p_i^1.

The atomic T-formulae are the expressions of the form $d_i\Phi$, where Φ is a T-formula. If Φ is an elementary T-formula, then $d_i\Phi$ will be called an elementary D-formula. The language $JLDn$ is the smallest set comprising all the atomic D-formulae, and closed under the operations of joining the expressions by means of the disjunction and conjunction connectives, prefixing them with the negation connective and putting them in brackets.

Let Φ and Ψ be two elementary D-formulae in the language $JLDn$. I shall say that Φ contradicts Ψ provided that Φ has the form $d_ip_j^k$, and Ψ has the form $d_ip_j^m$, where $k \neq m$, or else Ψ has the form $d_mp_j^n$, where $i \neq m$.

Let $j_1, \ldots, j_r$ be a non-decreasing sequence of natural numbers. Elementary conjunction of the language $JLDn$, connected with the sequence $j_1, \ldots, j_r$ will be a conjunction of the elementary D-formulae which has the form

(E) $d_{i_1}p_{j_1}^{k_1} \wedge \ldots \wedge d_{i_r}p_{j_r}^{k_r}$, where $1 \leqslant i_1, \ldots, i_r \leqslant n$.

An elementary conjunction is compatible if there are no two contradictory elementary D-formulae to appear in it. Otherwise I shall speak of the incompatible elementary conjunctions. The elementary conjunctions which are connected with the same sequence will be called non-equivalent provided that none of them may be obtained from the other by commuting the factors or deleting the factors that are equiform or, finally, by adding the factors equiform with those that have already occurred.

An arbitrary disjunction of elementary conjunctions of the language $JLDn$ connected with a non-decreasing sequence $j_1, \ldots, j_r$ will be called a regular formula of the language $JLDn$ connected with that sequence.

A formula will be said to be cannonic provided that it is a regular formula in which there appear all compatible non-equivalent elementary conjunctions connected with the given sequence.

II. Rules of transformation. The concept of thesis

Formulae of the language $JLDn$ are transformed with the help of the following six rules, where Φ is considered to be the only premise of the given rule while Ψ is the conclusion of this rule.

R1. If the expression $(\sim \Phi \vee \Psi) \wedge (\sim \Psi \vee \Phi)$ is a substitution of a tautology of the classical sentential calculus, then we may pass from Φ to Ψ.

R2. If the expression $(\sim \Phi \vee \Psi) \wedge (\sim \Psi \vee \Phi)$ is a thesis of the logic of change described in detail in the paper of T. Kubiński "Kryterium matrycowe dla logiki zmiany von Wrighta" (Matrix criterion for von Wright's logic of change, 1971), if Φ has the form $d_i\Phi_1$, and moreover Ψ has the form $d_i\Psi_1$, then we may pass from Φ to Ψ.

R3. If Φ has the form $d_i(\Phi_1 \wedge \Psi_1)$ and Ψ has the form $d_i(\Phi_1) \wedge d_i(\Psi_1)$, then we may pass from Φ to Ψ and vice versa.

R4. If Φ has the form $d_i(\Phi_1 \vee \Psi_1)$ and Ψ has the form $d_i(\Phi_1) \vee d_i(\Psi_1)$, then we may pass from Φ to Ψ and back.

R5. If the formula $\sim d_i p_j^k$ is a part (possibly improper) of the formula Φ, while Ψ is obtained from Φ by replacing $\sim d_i p_j^k$ at least in one place by the formula

$$\sum_{k=0}^{3}\sum_{m=1}^{i-1} d_m p_j^k \vee \sum_{k=0}^{3}\sum_{m=i+1}^{n} d_m p_j^k \sum_{m \neq k_{0 \leqslant m \leqslant 3}} d_i p_j^m,$$

then we may pass from Φ to Ψ and back.

R6. If the formula Φ_1 is a part (possibly improper) of the formula Φ, while Ψ is obtained from Φ by replacing Φ_1 at least in one place by the formula $\Phi_2 = \Phi_1 \wedge \sum_{k=0}^{3}\sum_{i=1}^{n} d_i p_j^k$, then we may pass from Φ to Ψ and back.

Let Φ be a formula in the language $JLDn$. Φ is a thesis of the logic of deeds LDn (in symbols: $LDn \vdash \Phi$) if there exists a sequence of formulae $\Phi = \Phi, \ldots, \Phi_n$ (called the proof of the formula) such that Φ_n is a cannonic formula and Φ_{i+1} is obtained from Φ_i (for $i < n$) by making use of one of the rules R1. – R6.

The following theorems about logics of deeds and their languages may be proved:

THEOREM 1. *Every formula Φ in the language $JLDn$ may be transformed* *(with the help of the rules R1. – R6.) into a regular formula in which there would occur all and only these variables in which occur in the formula* Φ.

THEOREM 2. *Every substitution of a tautology of the classical sentential calculus in the language* $JLDn$ *is a thesis of the logic of deeds* LDn.

THEOREM 3. *Every system* LDn *is decidable. The decidability procedure is based on the above stated rules of transformation.*

THEOREM 4. *If* m *and* n *are two distinct natural numbers greater than zero, then none of the following three sets is empty:* $LDm - LDn$, $LDn - LDm$, $LDm \cdot LDn$. *To put it in the other words: the logics of deeds* LDm *and* LDn *intersect.*

THEOREM 5. *If* m *and* n *are two distinct natural numbers greater than zero and if* m *is smaller than* n, *then* $JLDm$ *is a proper subset of the set* $JLDn$.

The infinite sequence $JLD1, JLD2, \ldots$ is then an increasing one, and the family of all languages $JLDn$ is monotonic.

The system $LD2$ is very similar to von Wright's logic of action.

III. Semantics for logics of deeds. n-valid formulae

I shall proceed now to discussing the semantics for the logic of deeds LDn. Let k be an arbitrary cardinal number greater than zero, and let Z be the set of the power $2k$. The set Z will be called the set of events. Let i be an involution without fixed points defined on the set Z. Every set composed of the following four elements, each of them being an ordered pair: $\langle a, a\rangle$, $\langle a, i(a)\rangle$, $\langle i(a), a\rangle$, $\langle i(a), i(a)\rangle$, where a is an element of the set Z, will be called a normal set (generated by the event a). Let S be a set in Z^2 such that it has the following property: for any normal set N, the set-theoretic intersection $s \cdot N$ is a unit set. Such set S will be called a selection in the set of events Z. Let c_1S and c_2S be respectively, cylindrfications of the selection S with regard to the first and with regard to the second expression of the ordered pairs. The set $u = c_1S \cdot c_2S$ is said to be the extension of the selection S.

A sequence $f_1, \ldots, f_n$ of functions will be called a sequence over the extension u provided that if $(\langle a, b\rangle) \in u$, then for exactly one function f_i

$f_i(\langle a,b\rangle) = 1$, while $f_j(\langle a,b\rangle) = 0$ for $j \neq i$, and if $\langle a,b\rangle \in Z^2 - u$, then $f_i(\langle a,b\rangle) = 0$ for all i.

If, in particular, $n = 1$, then f_1 is a characteristic function of the extension u. In the sequence composed of the functions f_1, f_2 over the extension u f_1 may be the function constants (i.e. taking the calue 0 for all arguments), while f_2 may be the characteristic function of the extension u.

Every sequence of the form $\langle Z, i, u, f_1, \ldots, f_n\rangle$ where $Z, i, u, f_1, \ldots, f_n$ have the sense specified above, will be called a simple sequence. If Z is a finite set, then the sequence will be called finite. Otherwise I shall speak of the infinite sequence.

Let $F_n = \langle Z, i, u, f_1, \ldots, f_n\rangle$ be an arbitrary simple sequence. Every function v defined on the alphabet of the logic of deeds LDn and taking the following values: $v(p_i)$ is an element of the set Z, $v(-) = i$, $v(T) = u$, $v(d_n) = f_n$, for $1 \leqslant k \leqslant n$, will be called a valuation in the sequence F_n. (It is apparent that in the language $JLD1$, the function v assigns the function f_1 to the constant d_1, where f_1 is a characteristic function of the set u).

Let Φ and Ψ belong to the language $JLDn$. I shall say that Φ and Ψ are semantically equivalent provided that for any simple sequence F_n and for any valuation v in F_n, either both formulae are satisfied in F_n under the valuation v, or none of them is satisfied under this valuation.

Let Φ be an element of $JLDn$. The formula Φ will be called satisfied under the valuation v in the simple sequence $\langle Z, i, u, f_1, \ldots, f_n\rangle$ if the valuation v assigns the number 1 to the formula Φ. The connectives of disjunction, conjunction and negation are given in their classical sense. The formula Φ will be called n-valid provided that Φ is satisfied in every simple sequence and under every valuation of this sequence. It is perhaps worth while to add that the formula of the form $d_i(\Phi \vee \Psi)$ (formula of the form $d_i(\Phi \wedge \Psi)$, formula of the form $d_i(\sim \Phi)$) is satisfied in the simple sequence F_n under the valuation v if and only if $d_i\Phi$ is satisfied or $d_i\Psi$ is satisfied (both $d_i\Phi$ and $d_i\Psi$ is satisfied, formula $d_i\Phi$ is not satisfied) in F_n under v.

The set of all n-valid formulae will be denoted by $vr(n)$.

IV. Completeness theorem

The concepts of thesis and of n-valid formula of the logic of deeds which were introduced above, enable us to formulate the completeness theorem. There holds the following

Theorem (on completeness of the logic of deeds LDn). *Let Φ be an arbitrary element of the language $JLDn$. Then $LDn \vdash \Phi$ if and only if Φ is in $vr(n)$.*

The proof of the stated completeness theorem is based on the following three lemmas:

Lemma 1. *In the proof of a given formula all the formulae are semantically equivalent.*

Lemma 2. *If Φ is a cannonic formula of the language $JLDn$, then Φ is in $vr(n)$.*

Lemma 3. *If Φ is a formula of the language $JLDn$ and its proof does not end with a cannonic formula, then Φ is not in $vr(n)$.*

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