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THE PROOF OF Ł-DECIDABILITY OF LEWIS' SYSTEM $S5$

Let us first remind the notion of Ł-decidable system introduced in [4]. Let an arbitrary sentential calculus be given and let it be assumed S is the set of all meaningful expressions of this calculus.

Let moreover,

- A, R denote respectively the set of accepted axioms and the set of (primitive) rules of acceptance;
- A^{-1}, R^{-1} denote respectively the set of rejected axioms and the set of (primitive) rules of rejection;
- T denote the set of theses (accepted expressions) of the given calculus, i.e., the set of all these expressions which either belong to the set A, or else may be obtained from the expressions of the set A by the rules in R;
- T^{-1} denote the set of rejected expressions, i.e., the set of all these expressions which either belong to the set A^{-1}, or may be obtained from the expressions of the set A^{-1} by the rules of the set R^{-1}.

We say that a sentential calculus is Ł-decidable (decidable in the sense of Łukasiewicz) if and only if the following conditions are satisfied:

$$T \cup T^{-1} = S,$$

$$T \cap T^{-1} = \emptyset.$$

Then an Ł-decidable system has such property that an arbitrary meaningful expression is either accepted or it is rejected and, at the same time, exactly one or the two possibilities is realized.

There are familiar proofs of Ł-decidability for Aristotle's syllogistic [3], for a certain modal system of sentential calculus [2] for all many-valued logics of Łukasiewicz [1].

The system $S5$ of Lewis will be considered here in the following formalization. The subsequent formulas are accepted axioms:

1. $\vdash LCCpqCCqrCpr$
2. $\vdash LCpCNpq$
3. $\vdash LCCNppp$
4. $\vdash LCLpp$
5. $\vdash LCLCpqCLpLq$
6. $\vdash LCLpLLp$
7. $\vdash LCNLpLNLp$
8. $\vdash CLpp$

The only rejected axiom is:

9. $\vdash p$.

The primitive rules of acceptance of the system $S5$ are detachment and substitution while the primitive rules of rejection are the rules based on the following schemes:

$r_1^{-1}: \dfrac{\vdash \beta, \vdash C\alpha\beta}{\vdash \alpha}$

$r_2^{-1}: \dfrac{\vdash \alpha^*}{\vdash \alpha}$, where α^* is a substitution of α;

$r_3^{-1}: \dfrac{\vdash C\alpha\beta_1, \ldots, \vdash C\alpha\beta_n}{\vdash CL\alpha A(L\beta_1, \ldots, L\beta_n)}$, where $\alpha, \beta_1, \ldots \beta_n$

are arbitrary meaningful expressions in which the symbol L does not occur and the generalized disjunction A is defined as follows:

$A(\alpha_1) = \alpha_1$

$A(\alpha_1, \ldots, a_k) = CN\alpha_1 \ldots CN\alpha_{k-1}\alpha_k$

Theorem. *The system S5 is Ł-decidable.*

In the proof of Ł-decidability for the system $S5$ an essential use is made of the expressions in their normal forms.

References

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