Bulletin of the **S**ection of **L**ogic
Volume 1/1 (1972), pp. 35–36

reedition 2015 [original edition, pp. 35–37]

Wojciech Suchoń

ON NON EQUIVALENCE OF TWO DEFINITIONS OF THE ALGEBRAS OF ŁUKASIEWICZ

This paper was read to the Conference Logical Calculi, Warsaw, October 1971. It was also reported in May 1971 at the seminar of the Department of Logic, Jagiellonian University, held by Professor Stanisław J. Surma in Cracow. The full text of the paper will appear in Zeszyty Naukowe Uniwersytetu Jagiellońskiego, Prace z Logiki, zeszyt 7.

Starting with the notion of the n-valued algebra of Łukasiewicz, introduced by Gr. C. Moisil (Definition 1), we may introduce the notion of the symmetric algebra (Definition 2).

We establish the fact that both the three-valued algebras of Łukasiewicz and the centred algebras are symmetric.

The n-valued algebra of Łukasiewicz defined by G. Georgescu and C. Vraciu (Definition 3) is also a symmetric algebra – that is what we see from Theorem 1.

Definition 1. (cf. Gr. C. Moisil [2]). We shall say that the algebra $\langle L, \{\sigma_i\}_{l \leqslant i \leqslant n}\rangle$ is the n-valued algebra of Łukasiewicz provided that:

(i) L is a distributive lattice

and

(ii) the elements of the sequence $\{\sigma_i\}$ are endomorphisms which differ from each other and such that:

$M1$ $\sigma_i : L \to C(L) = \{x \in L$: there exists $\overline{x}$ such that $x \cup \overline{x} = 1$ and $x \cap \overline{x} = 0\}$

$M2$ $\sigma_i x \subset \sigma_{i+1} x$

$M3$ $\sigma_i \circ \sigma_j = \sigma_j$

$M4$ $\sigma_i 1 = 1$ and $\sigma_i 0 = 0$

$M5$ if $\sigma_i x = \sigma_i y$ for any $1 \leqslant i \leqslant n$, then $x = y$

Definition 2. For any n-valued algebra of Łukasiewicz, L being a symmetric algebra, it is both sufficient and necessary that for every $x \in L$, $W_x \neq \emptyset$.

($W_x = \{z \in L$: for every $1 \leqslant i \leqslant n$ there exists $\sigma_i z = \sigma_{n-1} x\}$).

Definition 3. (cf. G. Georgescu, C. Vraciu [1]) We shall say that the algebra $\langle L, \{\sigma_i\}_{l \leqslant i \leqslant n}, N\rangle$.

(i) L is a distributive lattice

and

(ii) N is an involution while the elements of the sequence $\{\sigma_i\}$ are endomorphisms different from each other and such that:

$V1$ $NNx = x$
$V2$ $\sigma_i x \subset \sigma_{i+1} x$
$V3$ $\sigma_i \circ \sigma_j = \sigma_j$
$V4$ $\sigma_i N x = N \sigma_{n-1} x$
$V5$ $N\sigma_i x \cup \sigma_i x = 1$ and $N\sigma_i x \cap \sigma_i x = 0$
$V6$ if $\sigma_i x = \sigma_i y$ for every $1 \leqslant i \leqslant n$, then $x = y$.

Theorem 1. *In order that L be a symmetric n-valued algebra of Łukasiewicz in the sense of Definition 1 it is necessary and sufficient that L be an n-valued algebra of Łukasiewicz in the sense of Definition 3.*

References

[1] G. Georgescu, C. Vraciu, *n-valent centred Łukasiewicz algebras*, **Revue Roumaine de Mathématiques Pures et Appliquées** 14 (1969), pp. 793–802.

[2] Gr. C. Moisil, **Łukasiewicz algebras**, Bucuresti 1968 (manuscript).

Department of Logic
Jagiellonian University
Cracow