Bulletin of the **S**ection of **L**ogic
Volume 20:3/4 (1991), pp. 83–84

reedition 2005 [original edition, pp. 83–84]

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OPERATORS ON BRANCHED QUANTIFIERS

As was first suggested in Mostowski's work on a "surface approach" to branched quantifiers [1], the semantic essentials of branched quantifiers can be summed up in associated structures $< A, E, D >$ where A is a set of universally quantified variables, E the set of existentially quantified variables, and D the set of dependencies of existential variables on universal ones. I call such structures Mostowski graphs.

In this paper, we first sharpen our understanding of the nature and role of Mostowski graphs in the theory of branched quantifiers. The Mostowski graphs which represent branched quantifiers are a proper subset of the set of Mostowski graphs. A fundamental result is that all and only those Mostowski graphs which have no 5-paths and 6-cycles as vertex-induced subgraphs are capable of representing branched quantifiers.

Having a structural characterization of the class of graphs which represent branched quantifiers enables us to look at operators on branched quantifiers in two ways: through the syntax of the branched quantifiers themselves, and through the structures of the associated graphs. Each perspective provides a separate source of ideas, and yet each serves as a constrain on the other.

In this paper, I define a number of fundamental operators on branched quantifiers, including two normal form operators, and show how they relate to operators on the associated graphs.

To illustrate how the two perspectives serve as constraints on one another, I gave three examples. One example is of seemingly natural purely syntactic transformation (namely, left-to-right reversal) on branched quantifiers which in fact does not correspond to any (semantically significant) operator, as can be seen from the fact there is no corresponding operator on graphs. Surprisingly, though this transformation is not itself a genuine

operator on branched quantifiers, it can be combined with another transformation on branched quantifiers (the inversion transformation, also not an operator, which interchanges existential and universal quantifiers) to give another genuine (and interesting) operator on branched quantifiers. The third example is of a seemingly natural transformation on graphs which in fact does not correspond to any transformation on branched quantifiers, as is shown by the fact that it sometimes produces graphs with 5-paths or 6-cycles as vertex-induced subgraphs.

The paper closes with a variety of conjectures which form the agenda for some future work.

References

[1] M. Mostowski, **Branched Quantifiers**, Rozprawy Uniwersytetu Warszawskiego, (Warsaw) 1991.

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