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DECIDABILITY RESULTS FOR CLASSES OF ORDERED ABELIAN GROUPS IN LOGICS WITH RAMSEY-QUANTIFIERS

This paper is to contributive to the model theory of ordered abelian groups (o.a.g. for short). The basic elements to build up the algebraic structure of the o.a.g-s are the archimedean groups: By Hahn's embedding theorem every o.a.g. can be represented as a subgroup of the Hahn-product of archimedean o.a.g.s.

Archimedean is not a first-order concept but there exists a first-order model theory of o.a.g.-s has to be developed inside the framework of regularly o.a.g-s.

Weispfenning [2] showed that these are essentially the ones that admit elimination of quantifiers in the language $\{+, -, 0, <, \equiv_n | n < \omega\}$ of o.a.g-s. (possibly extended by a set of constant symbols). Moreover, this quantifier elimination procedure is a basic tool for the model-theoretic investigations in this field.

We start to develop the model theory of o.a.g-s inside the framework of extended logics: *archimedean* is a $\mathcal{L}(Q_0^n)$-phenomenon Q_0^n being the "Ramsey quantifier" (in the $\aleph_0$-interpretation) introduced by Magidor & Malitz [1]. We generalize the quantifier elimination results mentioned above to $\mathcal{L}(Q_0^n)$ and $\mathcal{L}_0^{<\omega}$. Especially, we prove quantifier elimination results for classes of non-archimedean o.a.g.s.

Since all these quantifier-elimination procedures are effective, this yields the decidability of the respective theories.

References

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