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QUANTIFIERS DEFINABLE BY SECOND ORDER MEANS

The concept of generalized quantifiers, as defined by Lindström, for some purposes is too general. A bit more subtle concept would be useful, when discussing axiomatizability of logics with such quantifiers.

We say that a generalized quantifier Q is definable by second order means if and only if there is a second order formula $\phi(P_1, \ldots, P_n)$ with only free variables $P_1, \ldots, P_n$ such that $Qx_1 \ldots x_n(\phi_1, \ldots, \phi_n)$ is semantically equivalent to $\phi(\phi_1, \ldots, \phi_n)$ (– a result of substitution of ϕ_i in ϕ in a place of P_i defined in a natural way, for $i = 1, \ldots, n$). In such a case we also say that ϕ is a defining formula for Q.

Observe that:

(1) if a logic $L(Q)$ (that is a logic with Q as an additional quantifier) is interpretable in the second order logic then Q is definable by second order means,

(2) practically all considered in literature Lindström quantifiers are definable by second order means.

Passing to weak semantics for the second order logic we consider structures of a form (M, K), where M is a standard structure and K is a class of relations over $|M|$. Interpretation of quantifiers definable by second order means in weak structures could depend on the choice of defining formulae.

Fixing defining formulae for Lindström quantifiers $Q_1, \ldots, Q_n$ we give a general method of defining natural axiomatic approximations of $L(Q_1, \ldots, Q_n)$. Moreover these approximations can be proved to be complete relatively to proper weak semantics (a class of structures (M, K) such that K is closed on definability over (M, K) by formulae of a considered language).

As examples of this general construction we can give the proof systems LB and LS (see M. Mostowski 1987 and 1991).

References

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