Bulletin of the **S**ection of **L**ogic
Volume 20:3/4 (1991), pp. 115–115

reedition 2005 [original edition, pp. 115–115]

Alexandre G. Pinus

GENERALIZED QUANTIFIERS IN ALGEBRA

Three extensions of the first order predicate calculus are considered. They are obtained by adding to a language the following quantifiers: A,$^\sim$, $<>$. These quantifiers reflect some properties of the theory of algebraic systems connected with concepts of automorphism, congruence and subalgebra.

If $\phi(x,\underline{y})$ and $\psi(x,\underline{y})$ are formulas of $L_{\omega,\omega}(A)$ (or $L_{\omega,\omega}(^\sim), L_{\omega,\omega}(<>)$ respectively) then $Ax(\phi(x,\underline{y}),\psi(x,\underline{y}))$ (or $^\sim x(\phi(x,\underline{y}),\psi(x,\underline{y}))$, $< x >$ $(\phi(x,\underline{y}),\psi(x,\underline{y}))$ respectively) is also a formula of $L_{\omega,\omega}(< A >)$ (or $L_{\omega,\omega}(^\sim)$, $L_{\omega,\omega}(<>)$ respectively).

If $\underline{a},\underline{b}$ are elements of an algebraic system $\mathcal{A}$ then:

$\mathcal{A} \models Ax(\phi(x,\underline{a}),\psi(x,\underline{b}))$ iff there is $f \in Aut(U)$ such that $f(\phi(U,\underline{a})) = \psi(U,\underline{b})$;

$\mathcal{A} \models\ ^\sim x(\phi(x,\underline{a}),\psi(x,\underline{b}))$ iff $\theta_U(\phi(U,\underline{a})) \subseteq \theta_U(\psi(U,\underline{b}))$, where $\theta_U(X)$ is a congruence generated on U by a set X;

$\mathcal{A} \models <x> (\phi(x,\underline{a}),\psi(x,\underline{b}))$ iff $<\phi(U,\underline{a})> \subseteq <\psi(U,\underline{b})>$, where $<X>$ is a subsystem of U generated by a set X;

The following questions related to languages $L_{\omega,\omega}(A), L_{\omega,\omega}(^\sim)$, and $L_{\omega,\omega}(<>)$ will be considered:

– Löwenheim and Hanf numbers,

– axiomatizability and compactness,

– expressive power of these languages,

– decidability of theories in these languages (such as: boolean algebras, abelian groups, algebraic closed fields, S-free algebras, and others),

– relations of these languages with infinitary and second order languages.