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A SEQUEL TO HAWRANEK/ZYGMUNT

In [1], taking a non-empty-set L, a non-empty collection of its subsets $\mathcal{R}$, and setting for an $M \subset L$:

$$r(M) = \{R \in \mathcal{R} : R \cap M \neq \emptyset\}$$

$$\mathcal{B} = \{B \subset M : r(B) = r(M)\},$$

the authors have shown (cf. their proof of "Proposition 2") that for any $B_0 \in \mathcal{B}$

$$B_0 \text{ is minimal in } (\mathcal{B}, \subset) \text{ iff } \bigwedge_{b \in B_0} \bigvee_{R \in \mathcal{R}} R \cap B_0 = \{b\},$$

and claimed this to be answer to the question put forward in [2]. We have a comment on that.

Henceforth take L, as in [2], to be a join-semilattice with unit, and $\mathcal{R}$ – the totality of its maximal ideals ("realizations"). Setting $A \vdash B$ iff $A \subset R \Rightarrow B \cap R \neq \emptyset$, for all $R \in \mathcal{R}$, the relation $\vdash$ is one of *entailment*. Let $A \nvdash B$ be its negation, and $x \vdash B$ short for $\{x\} \vdash B$. Clearly, for any $B \in \mathcal{B}$:

$$\bigwedge_{b \in B} \bigvee_{R \in \mathcal{R}} R \cap B = \{b\} \text{ iff } \bigwedge_{b \in B} b \nvdash (B - \{b\}), \tag{2}$$

which in view (1) yields another criterion of B's minimality in $(\mathcal{B}, \subset)$.

However, neither (1), nor (2), is an answer to our question. For that was for a criterion as to when a join-semilattice L is such that every $\mathcal{B}$-like collection of its subsets contains minimal members – by whatever criterion we come to recognize them.

References

[1] J. Hawranek, J. Zygmunt, *Comments on a question of Wolniewicz's*, **Bulletin of the Section of Logic** vol. 19, no. 4 (1990).

[2] B. Wolniewicz, *A question about Join-Semilattices*, **Bulletin of the Section of Logic** vol. 19, no. 3 (1990).

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