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A. G. Dragalin

THE COLLAPSE OF THE DESCRIPTIVE COMPLEXITY OF TRUTH DEFINITIONS. COMPLETIONS OF HEYTING AND BOOLEAN ALGEBRAS

We are interested in applying constructive methods in (classical and intuitionistic) model theory. We describe in a canonical way and in the frame of constructive metamathematics a procedure for an embedding with a small descriptive complexity of arbitrary Boolean (or Heyting) algebra in a *complete* algebra, preserving all existing unions and meets. As a prehistory of the method we mention the work of Friedman (as it described in [1]) and the well-known lemma of Rasiowa-Sikorski about completing of Boolean algebras with preserving a given *countable* family of unions and meets.

As an applications we get the following results about truth definitions with small descriptive complexity.

Theorem 1. *Let T be a r.e. classical axiomatic theory. A complete Boolean algebra B (whose elements are sets of natural numbers) can be constructed and a B-valued model $M(B)$ can be defined (in a constructive way) such that:*

(i) $n \in ||A||_B$ *is* Π^0_2 *predicate of* n *and* A*;*

(ii) $||A||_B = 1$ *iff* $T \vdash A$.

Theorem 2. *Let T be a r.e. intuitionistic axiomatic theory. A complete Heyting algebra H can be constructed and an H-valued model $M(H)$ can be defined (in a constructive way) such that:*

(i) $n \in ||A||_H$ *is* Σ^0_1 *predicate of* n *and* A*;*

(ii) $||A||_H = 1$ *iff* $T \vdash A$.

Let us consider now second order Peano arithmetic PA_2.

THEOREM 3. *A complete Boolean algebra B_1 and corresponding B_1-valued model $M(B_1)$ for PA_2 can be defined such that:*

(i) $n \in ||A||_{B_1}$ *is* Π^1_1 *predicate of* n *and* A*;*

(ii) $||A||_{B_1} = 1$ *is a* Π^1_1 *predicate of* A*;*

(iii) $M(B_1)$ *is standard on natural numbers (i.e.* $||\forall x A(x)|| = \bigwedge_{n\in\omega} ||A(x)||$ *for all formulas* A*) and* $M(B_1)$ *is complete and correct for the elementary truth definition;*

(iv) all sets in this model are definable in $M(B_1)$*.*

References

[1] A. S. Troelstra, D. van Dalen, **Costructivism in Mathematics. An Introduction**, vol. II North-Holland (1988).

[2] A. G. Dragalin, *A completeness theorem for higher-order intuitionistic logic; an intutionistic proof*, [in:] **Mathematical Logic and its Applications** (ed. Dimiter G. Skordev), Plenum Press, New York, (1987), pp. 107–124.

Mathematical Institute
University of Debrecen
Debrecen 4010, Pf. 12
Hungary