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ON MUTUAL NON-RECONSTRUCTABILITY OF THE ŁUKASIEWICZ CALCULI AND THEIR DUAL COUNTERPARTS

This is an abstract of the lecture presented at Seminar of the Section of Logic, Polish Academy of Sciences, December 1972.

To follow the arguments below it is necessary to have read the references [2] (for notion of reconstructability) and [1,3] (for the notion of the dual counterpart of a consequence) from which papers, also, all unexplained notations will come.

Theorem 1. *$(L, C^*_{\overline{k}})$ is not reconstructible in (L, C^*_k), if $k > 2$.*

Proof outline. If $(L, C^*_{\overline{k}})$ was reconstructible then some formula, say $\beta(p)$, would be a translation of the negation. Let us denote $\beta(p)$ by $⫬\ ⫬p$. Because it is the case that $C^*_{\overline{k}}(p, p) \neq L$, then $⫬\ ⫬1 = 1$. Hence the rule based on the schema

$$\frac{\alpha}{⫬\ ⫬\alpha}$$

would be a rule of C^*_k, which is not the case for $C^*_{\overline{k}}$, i.e. the rule based on the scheme

$$\frac{\alpha}{\sim \alpha}$$

is not valid for $C^*_{\overline{k}}$. The contradiction just obtained concludes the proof.

Theorem 2. *$(L, C^*_{\overline{k}})$ is not reconstructible in $(L, C^*_{\overline{k}})$, if $k > 2$.*

Proof outline. Assume the theorem to be false, and let the formula $\beta(p)$ (denoted in what follows by $⫬\ ⫬p$) be a translation of the negation sign. On the grounds of the following formulas:

(i) $C^*_k(p, \sim p) = L$ and

(ii) the rule based on the scheme $\frac{\alpha}{⫬\ ⫬\alpha}$ is valid for C^*_k, we can obtain that $\neg p$ ha sthe following truth-table:

(A)

p	p
0	1
$\frac{1}{k-1}$	1
.	.
.	.
.	.
$\frac{k-2}{k-1}$	1
1	$\neq 1$

Now from the formula

$$C^*_k(p) \cap C^*_k(\sim p) \neq C^*_k(\emptyset)$$

it follows that there is some formula $\alpha \in L$ with

(B) $\alpha \in C^*_k(p)$ and $\alpha \in C^*_k(\neg p)$ and $\alpha \notin C^*_k(\emptyset$.

But from (B) we can rather easily conclude that $\neg 1 = 1$, which is a contradiction with respect to (A).

The proofs just sketched are be used for the matrix version of infinite-valued Łukasiewicz calculus, too. Then we can have

THEOREM 3. *Neither* $(L, C^*_{\overline{\aleph_0}})$ *is reconstructible in* $(L, C^*_{\aleph_0})$ *nor* $(L, C^*_{\aleph_0})$ *is reconstructible in* $(L, C^*_{\overline{\aleph_0}})$.

References

[1] G. Malinowski and M. Spasowski, *Dual counterparts of Łukasiewicz sentential calculi*, this **Bulletin**, vol 1 (1972), no. 3, pp. 2–7.

[2] M. Tokarz and R. Wójcicki, *The problem of reconstructability of propositional calculi*, **Studia Logica**, vol. XXVIII (1971), pp. 119–127.

[3] R. Wójcicki, *Dual counterparts of consequence operations*, this **Bulletin**, vol. 2 (1973), no. 1.

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