Bulletin of the **S**ection of **L**ogic
Volume 2/3 (1973), pp. 195–196
reedition 2014 [original edition, pp. 195–196]

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ON DEFINING MOISIL'S FUNCTORS IN n-VALUED ŁUKASIEWICZ PROPOSITIONAL LOGIC

This paper was presented at the Seminar of the Department of Logic, Jagiellonian University in Cracow, held by Prof. Stanisław Surma, December 1972.

Gr. C. Moisil defined in his paper [1] the notion of n-valued Łukasiewicz algebra introducing a family of unary functions $\sigma_1^n, \ldots, \sigma_{n-1}^n$, which can be interpreted in sentential calculus as a special kind of modal functors:

$$v(\sigma_k^n \alpha) = \begin{cases} 1 & \text{if } v(\alpha) \geqslant \frac{k}{n-1} \\ 0 & \text{otherwise.} \end{cases}$$

In what follows we define the functions $\sigma_1^n, \ldots, \sigma_{n-1}^n$ in the n-valued Łukasiewicz propositional calculus (with implication and negation as the only connectives). It may be shown that

$$\sigma_k^n = N\sigma_{n-k}^n N,$$

which makes it possible to restrict ourselves to the construction of σ_i^n for $i \leqslant \frac{n}{2}$ only.

The construction will be carried ont on the grounds of a sequence of formulas defined inductively this way:

$$A_3(\alpha) = CN\alpha\alpha$$

$$A_{+1}(\alpha) = CN\alpha A_n(\alpha).$$

A very important property of A_n's can be formulated by means of the equality

$$v(A_m(\alpha)) = min(v(\alpha) \cdot (m-1), 1).$$

By virtue of this equality it is possible to prove that the functors σ_i^n can be defined as follows:

$$\sigma_1^n \alpha =_{df} A_n(\alpha),$$
$$\sigma_k^n \alpha =_{df} \sigma_s^n A_{m+1}(\alpha) \quad (\text{for } 1 < k \leqslant \tfrac{n}{2}),$$

where

$$m = max(j : j \cdot (k-1) \leqslant n-1)$$

and

$$s = \begin{cases} n-1 & \text{if } \ km \geqslant n-1, \\ km & \text{otherwise.} \end{cases}$$

References

[1] Gr. C. Moisil, *Notes sur les logiques nou chrysipiennes*, **Annales Scientifiques**, Jassy 27 (1941), pp. 80–98.

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