Bulletin of the **S**ection of **L**ogic
Volume 3/1 (1974), pp. 14–15

reedition 2012 [original edition, pp. 14–15]

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EN – SEMI – MODELS

This note is a summary of a part of author's doctoral dissertation supervised by Professors Roman Suszko ans Stephen L. Bloom, Department of Mathematics, Stevens Institute of Technology, Hoboken, N.J., 1973.

Let $\underline{A} = \langle A, -, \circ, \ldots \rangle$ be an algebra similar to the language of the EN-logic. The unary operation – and binary operation ∘ correspond to the connective of truth-functional negation and identity connective, respectively. Compare this Bulletin.

If $D \subseteq A$ and for all $a, b \in A$:

(1) $(-a) \in D$ iff $a \notin D$
(2) $(a \circ b) \in D$ iff $a = b$

then the pair $\langle \underline{A}, D \rangle$ is called (normal) EN-model. Notice that the condition (1) implies maximality of D. In fact if both D_1 and D_2 satisfy (1) and $D_1 \subseteq D_2$ then $D_1 = D_2$. Algebra $\underline{A}$ is said to be an EN-semi-model if there exists $D \subseteq A$ such that $\langle \underline{A}, D \rangle$ is an EN-model.

We will use the expression $-[k](a)$ to denote $\overbrace{- \ldots -}^{k} a$ for each $a \in A$ and $k = 0, 1, 2, \ldots$.

Theorem. *Algebra $\underline{A}$ is an EN-semi-model if and only if for all $a, b, c \in A$ and $i, j = 0, 1, 2, \ldots$:*

(3) $-[2i+1](a) \neq -[2j](a)$
(4) *if* $b \neq c$ *then both* $-[2i](a \circ a) \neq -[2j](b \circ c)$
and $-[2i+1](a \circ a) \neq -[2j+1](b \circ c)$

Corollary. *The class of all EN-semi-models is axiomatic with respect to the first order logic. On the other hand, the class of all EN-models is easily seen to be elementary with respect to that logic. For the notions involved see G. Grätzer, Universal Algebra, D. Van Nostrand 1968, chapter 42.*

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