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A NOTE ON INTUITIONISTIC SENTENTIAL CALCULUS (ISC)

Let FM be the set of all formulas of a sentential language with usual connectives $\neg, \wedge, \vee, \Rightarrow$ and $\Leftrightarrow$ and, let Ia be the set of logical axioms of ISC, see [1]. Every subset of FM containing IA and closed under the modus ponens rule is called a theory. $TAUT$ is the smallest theory. The quotient of FM modulo $TAUT$ is the free pseudo-Boolean algebra. By the prime filter theorem for distributive lattices: if α is not in the theory T then there exists a prime theory T_0 over T such that α is not in T_0.

Let TV be the set of all t in 2^{FM} such that for all α, β in FM:

(0) $t(\alpha) = 1$ whenever α is in IA
(1) $t(\alpha) = 0$ or $t(\neg\alpha) = 0$
(2) $t(\alpha \wedge \beta) = 1$ iff $t(\alpha) = t(\beta) = 1$
(3) $t(\alpha \vee \beta) = 0$ iff $t(\alpha) = t(\beta) = 0$
(4) either $t(\alpha \Rightarrow \beta) = 0$ or $t(\alpha) = 0$ or $t(\beta) = 1$
(5) $t(\alpha \Leftrightarrow \beta) = 1$ iff $t(\alpha \Rightarrow \beta) = t(\beta \Rightarrow \alpha) = 1$

Obviously, if T is any theory then $t(\alpha) = 1$ for every α in T and each t in TV. On the other hand, if α is not in the theory T then there exists t in TV such that $t(T) = 1$ and $t(\alpha) = 0$.

The intuitionistic consequence relation is defined for any subset X of FM and each α in FM as follows:

$$X \vdash \alpha \text{ iff } \alpha \text{ belongs to every theory over } X.$$

Theorem. *$X \vdash \alpha$ iff for all t in TV, $t(\alpha) = 1$ whenever $t(X) = 1$.*

Corollary. *The set $TAUT$ is decidable.*

Comment. Dr Basil Discord once told me that the ISC is a (logically) two-valued logic with truth-functional conjunction and disjunction and, non-truth-functional negation, implication and equivalence.

References

[1] H. Rasiowa and R. Sikorski, **The Mathematics of the Metamathematics**, PWN, Warszawa 1963.

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