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SOME NOTIONS AND THEOREMS OF MCKINSEY AND TARSKI AND *SCI*

Topological Boolean algebra (TB algebra) are considered here as described in [1].

TB algebra A is well connected iff (1) for all a, b in $A : a = b$ or $c = d$ whenever $(a \circ b) \cup (c \circ d) = 1$ (see [2]) or, equivalently, (2) A is a (normal) SCI-semi-model, i.e., there exists an ultrafilter U in A such that

$$a \circ b \text{ is in } U \text{ iff } a = b$$

Remark. In general, an algebra A similar to the SCI-language L is said to be a SCI-semi-model if there is a subset U of A such that $\langle A, U \rangle$ is a SCI-model.

Strong compactness of a TB-algebra (see [3]) implies well connectedness of it. The converse implication does not hold (M. Godlewski).

Given any TB algebra A and an arbitrary ultrafilter U in A, the relation holding between a and b iff

$$a \circ b \text{ is in } U$$

is a congruence of A and, the quotient A/U is well connected.

A SCI-theory T is said to be E-prime if for all $\alpha, \beta, \gamma, \delta$:

$$\text{if } (\alpha \equiv \beta) \vee (\gamma \equiv \delta) \text{ is in } T \text{ then}$$

$$\text{either } \alpha \equiv \beta \text{ is in } T \text{ or } \gamma \equiv \delta \text{ is in } T.$$

The SCI-theory WT also known as the modal system S_4 is E-prime (see [4]). The Lindenbaum-Tarski algebra L/WT, that is, the SCI-language L modulo the theory WT is the free TB algebra and, well connected as shown in [2].

THEOREM. *Let T be a consistent SCI-theory. Then, T is E-prime iff there exists a complete theory T_0 such that T is included in T_0 and both theories T and T_0 have the same equational kernel.*

COROLLARY. *Let T be an invariant equational E-prime SCI-theory. Then, the Lindenbaum-Tarski algebra L/T is a free (normal) SCI-semi-model. The corresponding equational class is defined by equations in T.*

WT is the smallest Boolean G-theory (i.e., closed under Goedel rule $\alpha, \beta/\alpha \equiv \beta$). Let T be a theory over WT. Then, T is equational iff T is a G-theory.

THEOREM. *Equational (and invariant) theories over WT constitute an inductive (or, algebraic) closure system whose basis is formed by finitely irreducible theories.*

THEOREM. *An equational (and invatiant) theory T over WT is E-prime iff T is finitely irreducible (to equational theories).*

QUESTION. What about equational Boolean theories properly included in WT?

References

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