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A CHARACTERIZATION OF FRAGMENTS OF THE INTUITIONISTIC PROPOSITIONAL LOGIC

We shall use the symbols: $\to$, $\leftrightarrow$, $\wedge$, $\vee$, $\neg$ as the well-known connectives (implication, equivalence, conjunction, disjunction, negation). For every set of connectives $Y \subseteq \{\to, \leftrightarrow, \wedge, \vee, \neg\}$ by F_Ψ we mean the set of formulas built up by means of propositional variables from an infinite set V and the connectives from Ψ (we shall write F instead of operation C in F_Ψ is called Ψ-consequence (see [1]) iff the following conditions hold for every $X \subseteq F_\Psi$, $\alpha, \beta \in F_\Psi$:

($\to$) if $\to \in \Psi$ then $C(X \cup \{\beta\}) \subseteq C(X \cup \{\alpha\})$ iff $\alpha \to \beta \in C(X)$,
($\leftrightarrow$) if $\leftrightarrow \in \Psi$ then $C(X \cup \{\beta\}) = C(X \cup \{\alpha\})$ iff $\alpha \leftrightarrow \beta \in C(X)$,
($\wedge$) if $\wedge \in \Psi$ then $C(\{\alpha, \beta\}) = C(\{\alpha \wedge \beta\})$,
($\vee$) if $\vee \in \Psi$ then $C(X \cup \{\alpha\}) \cap C(X \cup \{\beta\}) = C(X \cup \{\alpha \vee \beta\})$,
($\neg$) if $\neg \in \Psi$ then $C(X \cup \{\alpha\}) = F_\Psi$ iff $\neg\alpha \in C(X)$.

Let Cn denotes the consequence operation in F determined by the theorems of the intuitionistic propositional logic and the detachment rule for the implication connective $\to$. Putting $Cn_\Psi(X) = F_\Psi \cap Cn(X)$ for every $X \subseteq F_\Psi$ one defines the consequence operation Cn_Ψ in F_Ψ (obviously $Cn = Cn_\Psi$ in the case $\Psi = \{\to, \leftrightarrow, \wedge, \vee, \neg\}$). Grzegorczyk [1] proved that the intuitionistic consequence operation Cn can be characterized as the smallest $\{\to, \leftrightarrow, \wedge, \vee, \neg\}$-consequence. We have the following generalization of Grzegorczyk's results:

Theorem. *For every $\Psi \subseteq \{\to, \leftrightarrow, \wedge, \vee, \neg\}$, the consequence operation Cn_Ψ is the smallest Ψ-consequence.*

The fact above can be viewed as a kind of separable characterization of fragments of the intuitionistic propositional logic (comp. [2]). We are informed that a similar result was achieved independently by S. J. Surma (unpublished).

References

[1] A. Grzegorczyk, *An approach to logical calculus*, **Studia Logica** 30 (1972), pp. 33-41.

[2] A. Horn, *The separation theorem of intuitionist propositional calculus*, **J. S. L.** 27 (1962), pp. 391–399.

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