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CORRECTION TO MY NOTE "PARTIAL BOOLEAN σ-ALGEBRAS"

The Theorems 2a, 2b, 2c given in [1] are false.

Let $\underline{L} = \langle L; \overset{|}{\circ}; \vee, \neg; 1\rangle$ be a partial Boolean σ-algebra (or short: $\underline{L} \in PB\sigma A$) and $a \in L$. Then we put: $a^1 = a$, $a^0 = \neg a$. Let $(\varepsilon_1, \varepsilon_1, \ldots)$ denote any denumerable sequence in which ε_n equals 0 or 1 for every $n \in N$.

The author can only prove the following:

Theorem A. *Let $\underline{L} \in PB\sigma A$. Suppose $\underline{L}$ satisfies:*

For every denumerable sequence $(a_1, a_2, \ldots)$ of different elements of L there exists a sequence
$(*)$ $(\varepsilon_1, \varepsilon_2, \ldots)$ *such that for every subsequence $(a_{k_1}, a_{k_2}, \ldots)$ consisting of mutually commeasurable elements chosen from $(a_1, a_2, \ldots)$ the following inequality holds: $\bigwedge_{n\in N} a_{k_n}^{\varepsilon_{k_n}} \neq 0$.*

Then there exists a σ-homomorphism of $\underline{L}$ into a Boolean σ-algebra.

Theorem B. *Let $\underline{L} \in PB\sigma A$. Suppose $\underline{L}$ satisfies:*

For every denumerable sequence $(a_1, a_2, \ldots)$ of different elements of L and for every element a_{i_0} belonging to $(a_1, a_2, \ldots)$ there exists a sequence
$(**)$ $(\varepsilon_1, \varepsilon_2, \ldots)$ *such that*

1. *$\varepsilon_{i_0} = 1$ and*
2. *for every subsequence $(a_{k_1}, a_{k_2}, \ldots)$ consisting of mutually commeasurable elements chosen from $(a_1, a_2, \ldots)$ the following inequality holds: $\bigwedge_{n\in N} a_{k_n}^{\varepsilon_{k_n}} \neq 0$.*

Then $\underline{L}$ is weakly embeddable into a certain Boolean σ-algebra.

Theorem C. *Let* $\underline{L} \in PB\sigma A$. *Suppose* $\underline{L}$ *satisfies:*

For every denumerable sequence $(a_1, a_2, \ldots)$ *of different elements of L and for every two elements* a_{i_1}, a_{i_2} $(i_1 \neq i_2)$ *belonging to* $(a_1, a_2, \ldots)$ *there* $(***)$ *exists a sequence* $(\varepsilon_1, \varepsilon_2, \ldots)$ *such that*

1. *either* $\varepsilon_{i_1} = 1$, $\varepsilon_{i_2} = 0$ *or* $\varepsilon_{i_1} = 0$, $\varepsilon_{i_2} = 1$ *and*
2. *for every subsequence* $(a_{k_1}, a_{k_2}, \ldots)$ *consisting of mutually commeasurable elements chosen from* $(a_1, a_2, \ldots)$ *the following inequality holds:* $\bigwedge_{n \in N} a_{k_n}^{\varepsilon_{k_n}} \neq 0$.

Then $\underline{L}$ is embeddable into a certain Boolean σ-algebra.

Notice that if $\underline{L}$ satisfies $(**)$, then every Boolean sub-σ-algebra which is contained in $\underline{L}$ is σ-distributive.

References

[1] J. Czelakowski, *Partial Boolean σ-Algebras*, **Bulletin of the Section of Logic** of Inst. of Phil. and Soc. Pol. Acad. Sci., Vol. 3, No. 1.

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