Bulletin of the **S**ection of **L**ogic
Volume 42:3/4 (2013), pp. 169–181

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NON-ADJUNCTIVE DISCURSIVE LOGIC

Abstract

The discursive logic is sometimes treated as an example of the so-called non-adjunctive logic, which is correct but only as long as the definition of the discursive conjunction is deleted. In [1], we indicated a possible source of this confusion.

A non-adjunctive logic is not closed under the rule $P, Q \ / \ P \wedge Q$. From a practical viewpoint, it may be justly called an obstacle to be overcome at any rate.

Jaskowski's non-adjunctive discursive logic clearly shows the importance of a philosophical explanation for a formal approach to natural language. He points to the fact that people taking part in the discussion may disagree while being (self-) consistent. They may use some vague words or terms either purposefully or unintentionally. In such a case, "from the fact that a thesis P and a thesis Q have been advanced in a discourse it does not follow that the thesis $P \wedge Q$ has been advanced because it may happen that$P$ and Q have been advanced by different persons".[1]

The aim of this paper is to present a semantical study of the non-adjunctive discursive logic. We, additionally, show that the logic has a weak adjunctive property (in a sense given below).

Keywords: discursive (discussive) logic, paraconsistent logic.

[1] [7], p. 49.

1. Introduction

Some authors regard the discursive logic as a foremost example of the so-called non-adjunctive logic. Here are a few samples to illustrate the point.

'Let us start with non-adjunctive systems, so called because the inference from A and B to A $\wedge$ B fails. The first of these to be produced was also the first formal paraconsistent logic. This was Jaskowski's discussive (or discursive) logic.'[2]

'A non-adjunctive logic is a logic in which one cannot conclude A from A $\wedge$ B. The first non-adjunctive logic, and also the first paraconsistent logic, ever proposed is the discussive (or discursive) logic of Jaskowski.'[3]

'... consider the failure of adjunction:

A,B / A $\wedge$ B

well-known in discussive logic. (The non-adjunctive nature of Jaskowski's system has often been remarked upon and is frequently considered a major obstacle to the fruitful application of his paraconsistent approach.)'[4]

'Non-adjunctive logical systems are those where the inference from A and B to A $\wedge$ B fails. As is indicated in (Priest-Tanaka, 200) the first of these systems to be produced was also the first formal *paraconsistent logic*. This was Jaskowski's *discussive* (or *discursive*) logic.'[5]

'In discursive reasoning, we expect that A $\wedge \sim$ A does not hold while both A and $\sim$ A do. This means that the so-called *adjunction*, e.i. from $\vdash$ A, $\vdash$B, to $\vdash$ A $\wedge$ B is invalid. Jaskowski's *discursive logic* (or *discussive logic* is the first formal *paraconsistent logic* for discursive reasoning, which is classified as a $non - adjunctive\ system$.'[6]

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[2] G. Priest and K. Tanaka, *Paraconsistent Logic*, **Stanford Encyclopedia of Philosophy**, available at http://plato.stanford.edu/entries/logic-paraconsistent/

[3] S. Schaffert, F. Bry, P. Besnard, H. Decker, S. Decker, C. Enguix, A. Herzig, *Paraconsistent Reasoning for the Semantic Web*, available at http://www.pms.ifi.lmu.de/publikationen/PMS-FB/PMS-FB-2005-42.pdf

[4] [4], p. 651

[5] H. A. Costa, *Non-Adjunctive Inference and Classical Modalities*, **Journal of Philosophical Logic**, 34, 2005, pp. 581–605.

[6] S. Akama, K. Nakamatsu, J.M. Abe , *Constructive discursive reasoning*, in R. Setchi, I. Jordanov, R.J. Howlett, L.C. Jain (eds.) **Knowledge-Based and Intelligent Information and Engineering Systems. KES'10 Proceedings of the**

In [1], we showed that this claim was correct as long as we did not introduce the connective of the discursive conjunction, *id est*,

$$P \wedge_d Q =^{df} P \wedge \Diamond Q$$

or

$$P \wedge_d Q =^{df} \Diamond P \wedge Q.$$

We also indicated a possible source of this confusion.

In this paper we focus on the *non-adjunctive property* of Jaśkowski's discursive logic.

Since the connective of conjunction plays the key role here, we introduce some definitions used frequently throughout this paper.

Let *var* be a non-empty denumerable set of all propositional variables $\{p_1, p_2, ...\}$. We define by induction the set of discursive formulas based on *var* as the smallest set $dFm(var)$ for which the following holds

(i) if $P \in var$ then $P \in dFm(var)$
(ii) if $P \in dFm(var)$ then $\sim P \in dFm(var)$
(iii) if $P, Q \in dFm(var)$ then $P \bullet Q \in dFm(var)$, where $\bullet \in \{\vee, \wedge, \rightarrow_d, \leftrightarrow_d\}$.

The symbols: $\sim$, $\vee$, $\wedge$, $\rightarrow_d$, $\leftrightarrow_d$ denote negation, disjunction, conjunction, discursive implication and discursive equivalence, respectively.

Jaśkowski's (non-adjunctive and adjunctive) discursive logic was defined by an interpretation in the language of $S5$ of Lewis. To put it in more technical terms: let nAD_2 stand for the non-adjunctive discursive logic, then

$$nAD_2 =^{df} \{P \in dFm(var) : \Diamond f(P) \in S5\}.$$

A translation function f of the language of nAD_2 into the language of $S5$ of Lewis is the function defined as follows:

(i) $f(p_i) = p_i$ if $p_i \in var$ and $i \in N$
(ii) $f(\sim P) = \sim f(P)$
(iii) $f(P \vee Q) = f(P) \vee f(Q)$
(iv) $f(P \wedge Q) = f(P) \wedge f(Q)$

14th international conference on Knowledge-based and intelligent information and engineering systems: Part III, Springer, 2010, pp. 200–206.

(v) $f(P \to_d Q) = \Diamond f(P) \to f(Q)$

(vi) $f(P \leftrightarrow_d Q) = (\Diamond f(P) \to f(Q)) \wedge (\Diamond f(Q) \to \Diamond f(P))$.[7]

The idea of the translation function traces back to the paper that Jaśkowski published in 1948. Among the connectives, there is no discursive conjunction. Jaśkowski defined it in the following way

(iv)* $f(P \wedge_d Q) = f(P) \wedge \Diamond f(Q)$.

It is worth noting that a slightly different definition of the discursive conjunction was given by Newton C. A. da Costa and Lech Dubikajtis in 1977 (cf. [2]), viz.,

(iv)** $f(P \wedge_d Q) = \Diamond f(P) \wedge f(Q)$.

This is the main difference between the adjunctive and the non-adjunctive (*version* of the) discursive logic. This feature bears a number of *side effects*.

Firstly, we have lost the elegant definition of the discursive equivalence

$$P \leftrightarrow_d^\star Q =^{df} (P \to_d Q) \wedge_d (Q \to_d P)$$

in favor of

$$P \leftrightarrow_d Q =^{df} (P \to_d Q) \wedge (Q \to_d \sim (P \to_d (p \wedge \sim p))).$$

Secondly, the formula $(p \wedge \sim p) \to_d q$ is valid in nAD_2 and one could even question whether it was right to speak of nAD_2 as an example of paraconsistent logic. Luckily, Jaśkowski himself dispelled the doubt.[8]

And last, but not least, nAD_2 is not closed under the rule

(AdR) $P, Q \ / \ P \wedge Q$

since $S5$ is not closed under $\Diamond P$, $\Diamond Q \ / \ \Diamond(P \wedge Q)$. nAD_2 is then an example of an *non-adjunctive* logic.

[7] See [7] p. 44, [9] p. 57.

[8] [7], p. 47.

2. Algebraization of nAD_2

Section 2 is devoted to a description of the algebraic approach to nAD_2.

Notice that discursive equivalence is a definable connective in nAD_2 and so is redundant.

DEFINITION 2.1. $P \leftrightarrow_d Q =^{df} (P \rightarrow_d Q) \wedge (Q \rightarrow_d \sim (P \rightarrow_d (p\wedge \sim p)))$.

The connective of conjunction can also be removed, *id est*

DEFINITION 2.2. $P \wedge Q =^{df} \sim (\sim P \vee \sim Q)$.

Now we can introduce the notion of $nAD_2 - discursive$ algebra.

DEFINITION 2.3. An algebra $\mathbf{A} = \langle A, -, \vee, \rightarrow_d, 0, 1\rangle$ is called to be nAD_2-discursive iff the following conditions hold

(i) the structure $A = \langle A, -, \vee, 0, 1\rangle$ is a Boolean algebra

for every elements $a, b \in A$:

(ii) $a \rightarrow_d b = \begin{cases} 1, & \text{if } a = 0 \\ b, & \text{if } a \neq 0. \end{cases}$

Let K_{nAD2} denote a class of all the discursive matrices of the form $K_{nAD2} =^{df} \{\langle \mathbf{A}, A - \{0\}\rangle : \mathbf{A}$ is a nAD_2-discursive algebra$\}$ and $h : dFm(var) \rightarrow \mathbf{A}$ be a homomorphism of the language nAD_2 onto nAD_2-discursive algebra.

THEOREM 2.1. *For every formula $P \in dFm(var)$ the following holds: P is valid in nAD_2 iff for every $m \in K_{nAD2}$ and $h \in Hom(dFm(var), \boldsymbol{A})$, $hP \neq 0$.*

PROOF. By definition of nAD_2, translation function f and Henle algebra.

3. Possible worlds semantics

In this section, we introduce a Kripke-type semantics for nAD_2.

A frame (nAD_2-frame) is a pair $< W, R >$, where W is a non-empty set of possible worlds and R is the equivalence relation on W. A model (nAD_2-model) is a triple $< W, R, v >$, where v is a mapping from propositional variables to the sets of worlds, $v : var \longrightarrow 2^W$. The satisfaction relation $\models_m$ is recursively defined as follows

(var) $x \models_m p_i$ iff $x \in v(p_i)$ and $i \in N$
$(\sim)$ $x \models_m \sim P$ iff $x \not\models_m P$
$(\vee)$ $x \models_m P \vee Q$ iff $x \models_m P$ or $x \models_m Q$
$(\wedge)$ $x \models_m P \wedge Q$ iff $x \models_m P$ and $x \models_m Q$
$(\rightarrow_d)$ $x \models_m P \rightarrow_d Q$ iff if $\exists_{y \in W}(xRy$ and $y \models_m P)$ then $x \models_m Q$.

A formula P is valid in nAD_2 iff for any model $< W, R, v >$, for every $x \in W$, there exists $y \in W$ such that: xRy and $y \models_m P$.

Notice that the accessibility relation defined on nAD_2-frames is precisely the same as in $S5$ and each item listed above, i.e. from (var) to $(\rightarrow_d)$, has its counterpart in the definition of the translation function; to be more exact

PROPOSITION 3.1. *For every formula $P \in dFm(var)$ the following holds: P is valid in nAD_2 iff $\Diamond f(P) \in S5$ iff $P \in nAD_2$.*

PROOF. By induction.

With Proposition 3.1 is we have introduced a free-from-translation semantics for nAD_2.

The definition of the nAD_2-model can be further simplified due to the properties of the accessibility relation defined on nAD_2-frames.

A model (nAD_2-model) can be seen as a pair $< W, v >$, where W is a non-empty set of possible worlds and a function, $v : dFm(var) \times W \longrightarrow \{1, 0\}$, is inductively defined:

$(\sim)$ $v(\sim P,\ x) = 1$ iff $v(P,\ x) = 0$
$(\vee)$ $v(P \vee Q,\ x) = 1$ iff $v(P,\ x) = 1$ or $v(Q,\ x) = 1$
$(\wedge)$ $v(P \wedge Q,\ x) = 1$ iff $v(P,\ x) = 1$ and $v(Q,\ x) = 1$
$(\rightarrow_d)$ $v(P \rightarrow_d Q,\ x) = 1$ iff $\forall_{y \in W}(v(P,\ y) = 0)$ or $v(Q,\ x) = 1$.

A formula P is valid in nAD_2 iff for any model $< W, R, v >$, there exists $y \in W$ such that $v(P,\ y) = 1$.

Jaśkowski gave a few practical hints on how to read off the validity of some nAD_2-formulas directly from a classical true-value analysis, viz.,

(1) assume that a formula P does not include constant symbols other than $\sim$, $\wedge$ and $\vee$. If P is valid in the classical propositional calculus CPC, then (i) P and (ii) $\sim P \rightarrow_d Q$ are also valid in nAD_2, where $Q \in dFm(var)$;

(2) suppose now that P contains, besides variables, at most the connectives $\rightarrow$, $\leftrightarrow$ and $\vee$. If P is valid in CPC, then P_d is valid in nAD_2, where P_d is obtained from P by replacing $\rightarrow$, $\leftrightarrow$, $\vee$ with $\rightarrow_d$, $\leftrightarrow_d$, $\vee$, respectively;

(3) assume that a formula P_d is valid in nAD_2. Then P is valid in CPC, where P is obtained from P_d by replacing $\rightarrow_d$ and $\leftrightarrow_d$ with $\rightarrow$ and $\leftrightarrow$, respectively.[9]

4. Labeled Tableaux for nAD_2

In what follows we will use *signed labeled formulas* such as $\sigma :: TP$ or $\sigma :: FP$, where σ is a label and TP and FP are some formulas prefixed with "T" or "F". Intuitively, $\sigma :: TP$ is read as "P is true at the world σ" and $\sigma :: FP$ as "P is false at the world σ". A *label* is a natural number. We call ρ *root label* and always assume that $\rho = 1$. A tableau for a labeled formula P is a downward rooted tree, where each of the nodes contains a signed labeled formula, constructed using the branch extension rules to be defined below.

Negation:

$$(\mathbf{T}\sim)\ \frac{\sigma :: T \sim P}{\sigma :: FP} \qquad (\mathbf{F}\sim)\ \frac{\sigma :: F \sim P}{\sigma :: TP}$$

Conjunction:

$$(\mathbf{T}\wedge)\ \frac{\sigma :: TP \wedge Q}{\begin{array}{c}\sigma :: TP\\ \sigma :: TQ\end{array}} \qquad (\mathbf{F}\wedge)\ \frac{\sigma :: FP \wedge Q}{\sigma :: FP | \sigma :: FQ}$$

Disjunction:

$$(\mathbf{T}\vee)\ \frac{\sigma :: TP \vee Q}{\sigma :: TP | \sigma :: TQ} \qquad (\mathbf{F}\vee)\ \frac{\sigma :: FP \vee Q}{\begin{array}{c}\sigma :: FP\\ \sigma :: FQ\end{array}}$$

The rules $(T\wedge), (F\vee), (F \sim)$ and $(T \sim)$ are linear; $(F\wedge)$ and $(T\vee)$ are branching.

[9]See [7] pp. 45-49.

The rules are exactly the same as in case of the classical propositional logic. What is really interesting, however, is the case of the discursive implication.

Discursive implication:

$$(\mathbf{T} \rightarrow_{\mathbf{d}})\ \frac{\sigma :: TP \rightarrow_d Q}{\sigma^* :: FP \mid \sigma :: TQ} \qquad\qquad (\mathbf{F} \rightarrow_{\mathbf{d}})\ \frac{\sigma :: FP \rightarrow_d Q}{\begin{array}{c}\tau :: TP\\ \sigma :: FQ\end{array}}$$

where for $(T \rightarrow_d)$ σ^* has been *already used* in the branch and for $(F \rightarrow_d)$ τ is a label that is *new* to the branch.

A branch of a tableau is closed if we can apply the rule:

$$(\mathbf{C})\ \frac{\begin{array}{c}\sigma :: TP\\ \sigma :: FP\end{array}}{\text{closed}}$$

otherwise the branch is open. A tableau is closed if all of its branches are closed, otherwise the tableau is open.

Hereditary rule:

$$(\mathbf{FH})\ \frac{\rho :: FP}{\sigma^* :: FP}$$

where ρ is a root label and σ^* is a label that has been *already used* in the branch.

The application of (FH) is always limited to root labels.

Let P be a formula. By a *tableau proof* of P we mean a closed tableau with $1 :: FP$.

THEOREM 4.1. *A formula P has a tableau proof iff P is valid in nAD_2.*

PROOF. Since P is valid in nAD_2 iff $P \in nAD_2$ iff $\Diamond f(P) \in S5$ (due to Proposition 3.1) it is enough to show that a formula P has a tableau proof iff $\Diamond f(P) \in S5$. Notice that the rules for negation, disjunction and conjunction coincide with their classical counterparts. The rules of $(\mathbf{T} \rightarrow_{\mathbf{d}})$, $(\mathbf{F} \rightarrow_{\mathbf{d}})$ and $(\mathbf{FH})$ can, respectively, be transformed into

$$(\mathbf{TI})\frac{\sigma :: T\ \Diamond f(P) \rightarrow f(Q)}{\sigma^* :: F\ f(P) \mid \sigma :: T\ f(Q)}$$

$$(\mathbf{FI})\ \frac{\sigma :: F\ \Diamond f(P) \to f(Q)}{\begin{array}{c}\tau :: T\ f(P)\\ \sigma :: F\ f(Q)\end{array}}$$

$$(\mathbf{F}\Diamond)\ \frac{\rho :: F\ \Diamond f(P)}{\sigma^* :: F\ f(P)}$$

where σ^* ia a label that has *already been used* in the branch, τ is a label that is *new* to the branch and ρ is a root label.

Here is an examples of the tableau proof for $(P \wedge \sim P) \to_d Q$.

EXAMPLE 1. Closed tableau for $(P \wedge \sim P) \to_d Q$.

(a)	$1 :: F(P \wedge \sim P) \to_d Q$	(start)
(b)	$1 :: F\Diamond((P \wedge \sim P) \to_d Q)$	(by definition of nAD_2), (a)
(c)	$1 :: F\Diamond(\Diamond(P \wedge \sim P) \to Q)$	((v), definition of the function f), (b)
(d)	$1 :: F(\Diamond(P \wedge \sim P) \to Q)$	(($\mathbf{F}\Diamond$), $S5$), (c)
(e)	$1 :: T\Diamond(P \wedge \sim P)$	(($\mathbf{FI}$), $S5$),(d)
(f)	$1 :: FQ$	(($\mathbf{FI}$), $S5$),(d)
(g)	$2 :: T(P \wedge \sim P)$	(($\mathbf{T}\Diamond$), $S5$),(e)
(h)	$2 :: TP$	(($\mathbf{T}\wedge$), $S5$), (b)
(i)	$2 :: T \sim P$	(($\mathbf{T}\wedge$), $S5$), (b)
(j)	$2 :: FP$	(($\mathbf{T}\sim$), $S5$), (e)
	Closed	(h),(j).

Now, apply the method described in [3]. Notice, however, that we work with natural numbers (instead of a nonempty sequence of natural numbers separated by dots).

The (FH)-rule plays the key role in some tableau proofs. The following example illustrates how important and significant it is. But, for this time, the example will be free from the definition of nAD_2 and the translation function f.

EXAMPLE 2. Closed tableau for $P \to_d (Q \to_d \sim (Q \to_d \sim P))$.

(a)	$1 :: FP \to_d (Q \to_d \sim (Q \to_d \sim P))$	(start)
(b)	$2 :: TP$	$(F \to_d)$,(a)
(c)	$1 :: FQ \to_d \sim (Q \to_d \sim P)$	$(F \to_d)$,(a)
(d)	$3 :: TQ$	$(F \to_d)$,(c)
(e)	$1 :: F \sim (Q \to_d \sim P)$	$(F \to_d)$,(c)
(f)	$1 :: TQ \to_d \sim P$	$(F \sim)$,(e)

1^{st} *branch*

(g) $3 :: FQ$	$(T \to_d)$,(f)
Closed	(C),(d),(g)

2^{nd} *branch*

(g)$'$ $1 :: T \sim P$	$(T \to_d)$,(f)
(h)$'$ $1 :: FP$	$(T \sim)$,(g)$'$
(i)$'$ $2 :: FP \to_d (Q \to_d \sim (Q \to_d \sim P))$	**(FH),(a)**
(j)$'$ $4 :: TP$	$(F \to_d)$,(i)$'$
(k)$'$ $2 :: FQ \to_d \sim (Q \to_d \sim P)$	$(F \to_d)$,(i)$'$
(l)$'$ $5 :: TQ$	$(F \to_d)$,(k)$'$
(m)$'$ $2 :: F \sim (Q \to_d \sim P)$	$(F \to_d)$,(k)$'$
(n)$'$ $2 :: TQ \to_d \sim P$	$(F \sim)$,(m)$'$

1^{st} *sub-branch*

(o)$'$ $3 :: FQ$	$(T \to_d)$,(n)$'$
Closed	(C),(d),(o)$'$

2^{nd} *sub-branch*

(p)$''$ $2 :: T \sim P$	$(T \to_d)$,(n)$'$
(q)$''$ $2 :: FP$	$(F \sim)$,(p)$''$
Closed	(C),(b),(q)$''$

5. Unsigned Labeled Tableaux for nAD_2

In this section we introduce *labeled formulas* such as $\sigma :: P$, where σ is a label and P is a formula. Each label can be seen as a natural number. The notation $\sigma :: P$ intuitively means "P holds in world σ". *nAD_2-tableau* is a tree of labeled formulas with root label ρ, where ρ is always equal to 1. All the nodes of a tree are obtained by the rules schematically described as follows.

Conjunction:

$$(\wedge)\quad \frac{\sigma :: P \wedge Q}{\begin{array}{c}\sigma :: P\\ \sigma :: Q\end{array}} \qquad\qquad (\sim\wedge)\quad \frac{\sigma :: \sim (P \wedge Q)}{\sigma :: \sim P \mid \sigma :: \sim Q}$$

Disjunction:

$$(\vee)\quad \frac{\sigma :: P \vee Q}{\sigma :: P \mid \sigma :: Q} \qquad\qquad (\sim\vee)\quad \frac{\sigma :: \sim (P \vee Q)}{\begin{array}{c}\sigma :: \sim P\\ \sigma :: \sim Q\end{array}}$$

Negation:

$$(\sim\sim)\quad \frac{\sigma :: \sim\sim P}{\sigma :: P}$$

Discursive implication:

$$(\rightarrow_{\mathbf{d}})\quad \frac{\sigma :: P \rightarrow_d Q}{\sigma^\star :: \sim P | \sigma :: Q}\ \text{(for } \sigma^\star \textit{ already used})$$

$$(\sim\rightarrow_{\mathbf{d}})\quad \frac{\sigma :: \sim (P \rightarrow_d Q)}{\begin{array}{c}\tau :: P\\ \sigma :: \sim Q\end{array}}\ \text{(for } \tau \textit{ new})$$

Closing rule:

$$(\mathbf{C})\quad \frac{\begin{array}{c}\sigma :: P\\ \sigma :: \sim P\end{array}}{\bot}$$

Hereditary rule:

$$(\mathbf{H} \sim)\quad \frac{\rho :: \sim P}{\sigma^\star :: \sim P}\ \begin{array}{l}\text{(for } \textit{root label } \rho)\\ \text{(for } \sigma^\star \textit{ used})\end{array}$$

A branch of nAD_2-tableau is *closed* if it contains $\bot$, otherwise it is open. A nAD_2-tableau is *closed* if all of the branches it contains are closed; otherwise it is open. A *tableau proof* of P is a closed tableau with $1 ::\sim P$.

The rules $(\sim \vee), (\sim \wedge_d)$ and $(\rightarrow_d)$ are branching just like $(T\vee), (F\wedge_d)$ and $(T \rightarrow_d)$ given in Section 3. The others are linear. We always restrict the usage of (H $\sim$) to the root label ρ.

THEOREM 5.1. *A formula P has a tableau proof iff P is valid in nAD_2 iff $P \in nAD_2$ iff $\Diamond f(P) \in S5$.*

Here is an example of a tableau proof of $\sim\sim P \rightarrow_d P$.

EXAMPLE 3. Closed tableau for the law of double negation.

(a)	$1 ::\sim (\sim\sim P \rightarrow_d P)$	(start)
(b)	$2 ::\sim\sim P$	$(\sim\rightarrow_d)$,(a)
(c)	$1 ::\sim P$	$(\sim\rightarrow_d)$,(a)
(d)	$2 :: P$	$(\sim\sim)$,(b)
(e)	$2 ::\sim (\sim\sim P \rightarrow_d P)$	(H $\sim$),(a)
(f)	$3 ::\sim\sim P$	$(\sim\rightarrow_d)$,(e)
(g)	$2 ::\sim P$	$(\sim\rightarrow_d)$,(e)
(i)	$\bot$	(d),(g).

6. Weak Adjunctive Property

The non-adjunctive discursive logic is not closed under the rule (AdR). We cannot conclude $P \wedge Q$ from P and Q (for any P, $Q \in dFm(var)$). A question arises at this point as to whether a weaker formulation of the rule is admissible, i.e. P and $\bullet Q$ entail $P \wedge \bullet Q$ (where P, $Q \in dFm(var)$ and $\bullet$ is a negation). A simple calculation reveals that the rule $P, \sim Q \ / \ P \wedge \sim Q$ (for any P, $Q \in dFm(var)$) is not an admissible rule in nAD_2. Is it then correct to say that nAD_2 has weak adjunctive property? Let us introduce some definitions before answering the question.

Definition 5.1. $\neg_d P =^{df} P \rightarrow_d \sim (p \vee \sim p)$.

Definition 5.2. $\sim_d P =^{df} \sim (\sim P \rightarrow_d \sim (p \vee \sim p))$.

$\neg_d P$ can be thought of as saying that P is *impossible* whereas $\sim_d P$ as telling us that P is *unnecessary.* We can now extend the semantics presented in Section 3 by the following conditions:

$(\neg_d)$ $v(\neg_d P,\ x) = 1$ iff $\forall_{y \in W}(v(P,\ y) = 0)$
$(\sim_d)$ $v(\sim_d P,\ x) = 1$ iff $\exists_{y \in W}(v(P,\ y) = 0)$

and add the extra tableaux rules (Section 4):

Negation $\neg_d$:

$(\mathbf{T}\neg_\mathbf{d})\ \dfrac{\sigma :: T\neg_d P}{\sigma^* :: FP}$ $\qquad (\mathbf{F}\neg_\mathbf{d})\ \dfrac{\sigma :: F\neg_d P}{\tau :: TP}$

Negation $\sim_d$:

$(\mathbf{T}\sim_\mathbf{d})\ \dfrac{\sigma :: T \sim_d P}{\tau :: FP}$ $\qquad (\mathbf{F}\sim_\mathbf{d})\ \dfrac{\sigma :: F \sim_d P}{\sigma^* :: TP}$

where σ^* is a label that has been *already used* in the branch (for $(\mathbf{T}\neg_\mathbf{d})$, $(\mathbf{F}\sim_\mathbf{d})$) and τ is a label that is *new* to the branch (for $(\mathbf{F}\neg_\mathbf{d})$, $(\mathbf{T}\sim_\mathbf{d})$).

Notice that the formulas:

$P \rightarrow_d (\neg_d Q \rightarrow_d (P \wedge \bullet Q))$, where $\bullet \in \{\sim, \sim_d, \neg_d\}$
$P \rightarrow_d (\bullet Q \rightarrow_d (P \wedge \sim_d Q))$, where $\bullet \in \{\sim, \sim_d, \neg_d\}$

are valid in nAD_2 and nAD_2 is closed under the rules:

(WAdR1) P, $\neg_d Q \ / \ P \wedge \bullet Q$, where $\bullet \in \{\sim, \sim_d, \neg_d\}$
(WAdR2) P, $\bullet Q \ / \ P \wedge \sim_d Q$, where $\bullet \in \{\sim, \sim_d, \neg_d\}$.

Although nAD_2 is an example of the non-adjunctive logic it has a weak adjunctive property.

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