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MODAL SENSE OF THE CLASSICAL CONCEPTS OF *REASON FOR THE EXISTENCE OF BEINGS**

Abstract

The purpose of this article is to defend the classical philosophical thinking against the myth of infallibility of contemporary *philosophical* logic, in which logic is more philosophical than its philosophy is logical. Apart from the well-known problems with the formalization of intensional language, it should be noted that the so-called "standard" formal languages used today to formalize philosophical views do not adequately code the nature of this transcendent reality in them (in relation to cognition) In particular, any transformation of the classical language into higher *rows* and *types* that were introduced to the logic by Bertrand Russell is out of the question. For classical philosophers, the world has always been a "zero-order" because the things, characteristics, situations or relatives do not form any hierarchy in their common meaning. The world does not have *features* or *situations* other than those specific to particular things, and *sets* are only *concepts*. In addition, the distinction between *individual* and *general* names introduced by John Stuart Mill, in turn, has brought confusion to the expectations that formal logic with name-variables can be used as applied logic (It must be settled empirically even whether a singular name is an individual or a general one, such as "Yahweh"). Even Stanisław Leśniewskis precise Ontology does not allow us to express anything in a natural way, for example that *Philosophus est homo* for as long as there are more philosophers than one. And in accordance with *a weak* interpretation of the scheme *SaP* - most often accepted by modern logicians, the phrase "every god exists" - paradoxically - should be true for atheists, obviously,

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because in their opinion the term "god" is empty. Therefore, we have to count on a greater understanding of the reader who thinks that the *sets* are things of the world, not just concepts *cum fundamento in re*, and generally all that is, is a category of the *type* specified in a *row* in the *pre-established hierarchy* of these rows.

The concept and principle of reason, and in particular a sufficient reason for being, plays a key role in the philosophy of nature and the existence of the absolute. And, as a part of classical philosophy (i.e.: Aristotelian-Scholastic-Leibnizian-neo-Thomistic[1]) it involves traditional language codes, which are concepts and judgments. They need to be translated into a new, more precise – and nowadays used in analytic philosophy – system of formalisms (calculi) enriched with set-theoretic semantics. The purpose of this presentation is to attempt to define the language of universals as statements about the sets.

At the beginning, let us assume that we speak of *all*, regardless of what it is and what it is like. And let variables α, β, γ, α_1, α_2,... represent "all" ("whatever"). The word "is" is the fundamental predicate of classical philosophy. It is ambiguous – and to say that "the ontic subject α has the attribute β", "the range of a concept ... is included in the range of the concept ...", "... = ...", "the ... is an element of the range of a concept ..." – and the choice of one meaning depends on context.

Let us adopt the abbreviations:

$$\begin{aligned} \beta(\alpha) &=: \alpha \text{ is } \beta \\ \beta(\alpha, \alpha_1, \alpha_2, \dots) &\leftrightarrow (\beta(\alpha) \wedge \beta(\alpha_1) \wedge \dots) \\ (\alpha, \alpha_1, \alpha_2, \dots) \in \beta &\leftrightarrow (\alpha \in \beta \wedge \alpha_1 \in \beta \wedge \alpha_2 \in \beta, \dots) \end{aligned}$$

Hence:

1. Whatever term $(\alpha, \beta, \gamma, \alpha_1, \alpha_2, \dots)$.
2. Inconsistent $(\alpha) \leftrightarrow \exists_\beta(\beta(\alpha) \wedge \neg\beta(\alpha))$. "For the same [property] cannot together belong and not belong to the same [object] and in the same respect." (Aristotle, M G 3 1005 b 19-20)
3. The abbreviation: 'Set (α)' = 'α is a set'.

A *set* ought to be distinguished from a *collection*, which is a collective set, or a non-empty non-contradictory set, whose elements are interrelated in

[1]The author has analyzed this type of philosophy, with the means of formal logic, for example, in the book: [9] and in articles: [5], [6], [7], [8], [10].

one consistent system. This consistent system of collection is a reason to include it actually in individuals.

When we call the family of all supersets of a set α (main filter of the inclusion generated by α) its *intension*, then the generalized product of any subset of this filter – when it coincides with the set α – is called the *essence* of the set. When any essence of a set α is empty, then the set α is contradictory.

Relatives, or combinations of objects in accordance with Aristotle's scheme: "*A* of *B*", "something of something" (An. Pr. I, 23. The third paragraph from the top)[2], i.e. 'α_1/α_2', deserve particular attention.

Here are some of the examples given by Aristotle: "All relatives have correlatives: by the term 'slave' we mean the slave of a master, by the term 'master', master of a slave"[3]. Similarly: *the wing of the bird*, *the wing of the winged*, *the rudder of the boat*, *the head of the animal*.

The relative 'β/α' =: 'beta/alpha'. For example: something/anything, God/Israel, essence/absolute, philosopher/law.

Assume β^+ =: 'betas'.

Thus, for example, "essence$^+$/absolute" is essences/absolute, "essence/absolute$^+$" is the essence of absolutes, "essence$^+$absolute$^+$/" is essences/absolutes.

4. $\text{Set}(\alpha) \rightarrow \text{Intension}/\alpha\,(\{\beta : \alpha \subseteq \beta\})$, intension of α is called the family of all its supersets (main filter of the inclusion generated by α).
5. $\text{Set}(\alpha) \wedge \text{Intension}/\alpha\,(\{\beta : \alpha \subseteq \beta\}) \wedge \text{Set}/\text{Set}^+\,(\Delta) \wedge \Delta \subseteq \{\beta : \alpha \subseteq \beta\}) \rightarrow [\text{Essence}/\alpha\,(\gamma) \leftrightarrow \gamma = \bigcap \Delta]$.

A generalized product of any subset of the main filter, when this filter coincides with the set of α, is called the essence of the set α.

6. $\alpha \subset \beta \leftrightarrow \text{Set}(\alpha, \beta) \wedge \alpha \subseteq \beta \wedge \alpha \neq \beta$.
7. Universals are sets. But not every set is Universal. No empty or inconsistent set is one.

$\text{Universal}(\alpha) \leftrightarrow \text{Set}/\text{Individual}^+(\alpha) \wedge \alpha \neq \neg\emptyset \wedge \neg\text{Inconsistent}(\alpha)$. It is a universal concept that is a non-empty and consistent set of individuals.

[2]See: [2].

[3](Cat. 7. The third paragraph from the top) in [1].

8. The abbreviation: '$\alpha < \beta$' = 'α is in a subsumption relation to the β'. 'α is subordinated (submitted) to β', "included in another as in a whole" (An. Pr. I, 1. The first paragraph from the bottom).
   $\alpha < \beta \leftrightarrow \text{Universal}(\alpha, \beta) \wedge \alpha \subset \beta$.
9. $\alpha \,\text{has}\, \beta \leftrightarrow \exists_\gamma(\neg\text{Inconsistent}(\gamma) \wedge \beta/\alpha(\gamma)\,)$.
10. $\text{PropertyOf}/\alpha(\beta) \;\leftrightarrow \alpha \,\text{has}\, \beta$
11. $\text{Species}/\beta(\alpha) \;\leftrightarrow \alpha < \beta, \beta$ has a species.
12. $\text{Genus}/\alpha(\beta) \;\leftrightarrow \alpha < \beta, \alpha$ has Genus.
13. $\text{Species}(\alpha) \leftrightarrow \exists_\beta \text{Species}/\beta(\alpha)$.
14. $\text{Genus}(\beta) \leftrightarrow \exists_\alpha \text{Genus}/\alpha(\beta)$.
15. $\text{Category}(\alpha) \leftrightarrow \text{Genus}(\alpha) \wedge \neg\text{Species}(\alpha)$.
16. $\text{Unit[Concrete, Singleton]}\ (\alpha) \leftrightarrow \text{Species}(\alpha) \wedge \neg\text{Genus}(\alpha)$.

Variables: $t, x, y, z, t_1, t_2, x_1, x_2, \ldots$ represent individuals, thus:

17. $\text{Individual}(t, x, y, z, t_1, t_2, x_1, x_2, \ldots)$
18. $\text{Species}(\{x\}) \wedge \neg\text{Genus}(\{x\})$.
19. $\text{Individual}(x) \wedge \neg\text{Unit}(x)$.

Example: Unit(the white figure here)$\wedge\neg$Individual(the white figure here).

20. However: Individual($Kalias$)$\wedge\neg$Unit($Kalias$), because it is no species.

In the object language, things are denoted by nouns and nominal functions, features/things – are denoted by adjectives, features/situations – by adverbs, and the situations – with the help of sentences and propositional functions.

21. $\text{Category}/x\,(k) \leftrightarrow \text{Category}(k) \wedge x \in k$.
22. $\text{Universal}(S) \to (\text{Category}/S(k) \;\leftrightarrow \forall_{x/S}\, x \in k)$.
23. In classical philosophy there was no habit of creating universals of mixed category, which is why we adopt a rule that:

$$\text{Universal}(\alpha) \to \forall_{x,k,l}(x \in \alpha \wedge \text{Category}/x(k, l) \;\to k = l)$$

24. $\text{Part}/y(x) \;\leftrightarrow \exists_\alpha(\text{Universal}(\alpha) \wedge \alpha = (\text{Part}^+/y\,) \wedge x \in \alpha)$.
25. $\text{Part}(x) \leftrightarrow \exists_y \text{Part}/y(x)$
26. $y \,\text{has Part}^+ \leftrightarrow \exists_x \text{Part}/y(x)$.
27. $\text{Simple}(x) \leftrightarrow \neg\text{Part}(x) \wedge \neg(x \,\text{has Part}^+)$
28. $\text{Atom}(x) \leftrightarrow \text{Part}(x) \wedge \neg(x \,\text{has Part}^+)$.
29. $\text{Set/Atom}^+/\alpha \;= \{x\colon \exists_\beta(\beta{\in}\{r, c, s\} \wedge \alpha \subseteq \beta \wedge x \in \alpha \wedge \text{Atom}(x))\}$.

Hence: Sets of atoms are: Set/$Atom^+/r^+$, Set/$Atom^+/c^+$, Set/$Atom^+/s^+$,

30. Aggregate/$\alpha(y)$ $\rightarrow$ Set/Individual$^+(\alpha)$ $\wedge \forall_x(x \in \alpha \rightarrow$ Part/$y(x)$).
31. $\forall_{k,y}$(Category/$y(k)$ $\rightarrow \forall_x$(Part/$y(x)$ $\rightarrow$ Category/$x(k)$)).

Hence, any part of a thing is a thing, any part of a feature is a feature and every part of a situation is a situation.

32. *Components* should be distinguished from parts.

Each category has a specific component such as its substrate, a material that does not fall into any category, but only "fills" it. For example, timber is a component of the tree, but is neither a tree nor a part of it.

33. Set/Atom$^+/x$ $\neq \emptyset \rightarrow$ Component/x $=$ Aggregate/Atom$^+/x$.
34. There are two types of extension: spatial and temporal. The extension is an essential attribute (not a feature but property[4]) of matter, and the time extension – of reality. At the same time, the material beings are always space-time ones, and real entities can also be non-spatial, but temporal, and even outside the space-time ones as it is proposed, for example, with regard to the absolute.
35. Transcendentals, also called the principles of being, are intentional extrapolation of categories. They are over-categorical aspects of all individuals and universals. Even the categoricity, the substrate of categorial items, any spatial or temporal extension, essence, existence, form, structure, etc., are all aspects of such categorial instances, but they themselves do not belong to any category, neither as its element, or an element of an item or as part of the latter. They are entitled to individuals in the same way as features, but being properties, they are not characteristics.
36. Form/x (Aggregate/Feature$^+/x$).
37. Structure/x (Aggregate/Situation$^+$/Part$^+/x$).

[4]According to the traditional philosophy and logic, the real definitions (by *genus* and *differentiam specificam*) can not be created for the summa genera, i.e. categories. Simplifying numerous versions of categories to three: *things*, *features* and *situations*, we do not undertake to define them also here, content with explaining their, with the help of equivalences: Feature/$\alpha(\beta) \leftrightarrow \beta(\alpha)$, Property/$\alpha(\beta) \leftrightarrow \alpha\mu\beta$, Situation$(\alpha) \leftrightarrow \alpha$. For the classical philosophers, the *existence of*, for example, is not a feature (or characteristic), but only a property (or proprietary) of things, because things are not any existence, but have it. E.g. The God's existence is *proprietas* but no *qualitas* (God has His existence but – excluding metaphors – is not this existence).

38. Formal Existence/α(Consistency/α) . Where,
    Consistency/α (Situation:$\neg\exists_\beta(\beta(\alpha) \wedge \neg\beta(\alpha))$).

Let the variable “T” represent the time as a collection of “moments”, “points of time”. Let the variable “P” represent the space as a set of “places”, “points of space”.

39. Time (T) $\rightarrow t, t_1, t_2, \dots \in T$.
40. Space (P) $\rightarrow p, p_1, p_2, \dots \in P$.

Shortcut: ‘$A_t(\alpha)$’ =:‘α is present (topical) at the moment t’, for $t \in T$. ‘$M_p(\alpha)$’ =:‘α is in the place p’, for $p \in P$. Shortcut: ‘$\Diamonddot A\alpha$’ =:‘α is sometimes present’ ‘$\Diamonddot$M(α)’ =: ‘there are places where α is’.

41. $\Diamonddot A\alpha \leftrightarrow \exists_t A_t(\alpha)$.
42. $\Diamonddot \mathrm{M}(\alpha) \leftrightarrow \exists_p M_p(\alpha)$.

Shortcut: ‘$\boxdot$A(α)’ =: ‘α is always present’. ‘$\boxdot$M(α)’ =: ‘α is everywhere’.

43. $\boxdot \mathrm{A}(\alpha) \leftrightarrow \forall_t A_t(\alpha)$.
44. $\boxdot \mathrm{M}(\alpha) \leftrightarrow \forall_p M_p(\alpha)$.
45. Classical philosophy postulates that:

$$\mathrm{Absolute}(\alpha) \wedge \mathrm{Form}/\alpha(\beta) \rightarrow (\boxdot \mathrm{M}(\alpha) \wedge \boxdot \mathrm{A}(\alpha) \wedge \boxdot \mathrm{A}(\beta) \wedge \\ \wedge \forall_\gamma(\mathrm{Form}/\alpha(\gamma) \rightarrow \gamma = \beta)).$$

It may be suggested to start a sentence: ‘As absolutely unchangeable is the absolute present at any time’. But it is probably so just for an observer, and objectively, in itself, as absolutely unchangeable it is timeless. Because time is only where there is movement.

46. $\mathrm{Real}(\alpha) \leftrightarrow \Diamonddot A\alpha$,
47. $\mathrm{Nonentity}(\alpha) \leftrightarrow \boxdot\neg \mathrm{A}(\alpha)$.

Of course, real nonentity can be formal entity.

48. $\mathrm{s}(\alpha) \rightarrow [\Diamond\alpha \leftrightarrow \neg\mathrm{Inconsistent}(\Diamonddot A\alpha)]$, where $\mathrm{s}(\alpha)$ =: α is a situation.
49. $\mathrm{s}(\alpha) \rightarrow [\Box\alpha \leftrightarrow \mathrm{Inconsistent}(\Diamonddot\neg \mathrm{A}(\alpha))]$.
50. Reason_of_the_Existence/β $(\alpha) \leftrightarrow \neg\Diamond(\Diamonddot A\beta \wedge \boxdot\neg \mathrm{A}(\alpha))$.
51. Sufficient_Reason_of_the_Existence/β $(\alpha) \leftrightarrow$ Reason_of_the_Existence/ $(\beta)\alpha \wedge \forall\gamma$(Reason_of_the_Existence/α $(\gamma) \rightarrow \gamma = \alpha$).
52. Principle/Reason_of_the_Existence
    ($\forall\beta\exists\alpha$ Reason_of_the_Existence/β (α)).

53. Principle/Sufficient_Reason_of_the_Existence
   ($\forall\beta\exists\alpha$ Sufficient_Reason_of_the_Existence/β (α)).

Formal ontology language generated by inspirations of classical philosophy

Vocabulary

1. Variables
   (a) Nominal variables:
      i. Variables representing individual names (their set is denoted by constant I): $(x, y, z, x_1, x_2, \dots) \in I$
      ii. Temporal variables: $t, t_1, t_2, \dots$ (forming a set of T): $(t, t_1, t_2, \dots) \in T$
      iii. Variables representing the general names (their set is denoted by constant G): $(f, g, h, g_1, g_2, g_3, \dots) \in G$
   (b) Propositional variables (the set V): $(p, q, p_1, p_2, p_3, \dots) \in V$
   (c) Predicate variables (binary): $R, R_1, R_2, \dots$ (the set Ξ): $(R, R_1, R_2, \dots) \in \Xi$
2. Constants
   (a) Logic constants:
      i. truth-functional operators: $\neg, \rightarrow, \leftrightarrow, \wedge, \vee$
      ii. modal operators are:
         - alethic operators: $\Box, \Diamond$
         - temporal operators: $\Diamond A \dots$ (it is sometimes the case that ...), $\boxdot$A(...) (it is always the case that. ...)
      iii. quantifiers: $\forall, \exists$
   (b) set-theoretical constants: $Z \dots$ (... is a set), $\dots \in \dots$
   (c) special constans:
      i. nominal constants denoting categories (forming a set of general names N): r (things), c (features), s (situations): $(r, c, s) \in N$
      ii. name-building functor of one nominal argument: '$-$' ("non-...")
      iii. sign of the relative: $\dots/\dots$ (something of anything)
      iv. predicates:

$=$	(. . . is identical with. . .), (. . . is covered with . . .)
$\sigma(\ldots)$	(. . . is inconsistent)
$A_t(\ldots)$	(. . . is present at the time t)
$\preccurlyeq \ldots$	(. . . is contained in . . .) the subsumption relation
$\ldots \sqsubset \ldots$	(. . . is a part of. . .)

(d) Round brackets

Syntax rules

(a) Terms
Set – Φ – of nominal expressions (of terms) is the smallest of the sets of expressions which satisfies the conditions of the inductive definition: the starting point 1(X) and induction conditions: 2(X) – 3(X):

$$\Phi = \bigcap X[1(\mathrm{X}) \wedge 2(\mathrm{X}) \wedge 3(\mathrm{X})]$$

where:

$$1(\mathrm{X}) \leftrightarrow I \cup G \cup T \cup N \subseteq X$$
$$2(\mathrm{X}) \leftrightarrow \forall_{\tau}(\tau \in X \rightarrow `-\tau\text{'} \in X)$$
$$3(\mathrm{X}) \leftrightarrow \forall_{\tau_1}\forall_{\tau_2}(\tau_1, \tau_2 \in X \rightarrow `\tau_1/\tau_2\text{'} \in X).$$

(b) Formulas
Set – Ψ – of propositional formulas of the language is the smallest of the sets of expressions which satisfies the conditions of inductive definition: the starting points 1(Y) – 7(Y) and induction conditions: 8(Y) – 10(Y):

$$\Psi = \bigcap Y[1(\mathrm{Y}) \wedge 2(\mathrm{Y}) \wedge 3(\mathrm{Y}) \wedge 4(\mathrm{Y}) \wedge 5(\mathrm{Y}) \wedge 6(\mathrm{Y}) \wedge 7(\mathrm{Y}) \wedge 8(\mathrm{Y}) \wedge \wedge 9(\mathrm{Y}) \wedge 10(\mathrm{Y})]$$

where:

$1(Y) \leftrightarrow V \subseteq Y$
$2(Y) \leftrightarrow \forall_\tau[\tau \in \Phi \rightarrow \text{`}\sigma(\tau)\text{'} \in Y]$
$3(Y) \leftrightarrow \forall_{\tau_1}\forall_{\tau_2}\forall_R(\tau_1, \tau_2 \in \Phi \wedge R \in \Xi \rightarrow \text{`}\tau_1 = \tau_2\text{'}, \text{`}\tau_1 R \tau_2\text{'} \in Y)$;
$4(Y) \leftrightarrow \forall_{\tau_1}\forall_{\tau_2}(\tau_1, \tau_2 \in G \cup N \rightarrow \text{`}\tau_1 \preccurlyeq \tau_2\text{'} \in Y)$;
$5(Y) \leftrightarrow \forall_\tau\forall_t[\tau \in G \cup I \cup N \wedge t \in T \wedge \neg\sigma(\tau) \rightarrow \text{`}A_t(\tau)\text{'}, \text{`}A(\tau)\text{'} \in Y]$;
$6(Y) \leftrightarrow \forall_{\tau_1}\forall_{\tau_2}(\tau_1, \tau_2 \in I \rightarrow \text{`}\tau_1 \sqsubset \tau_2\text{'} \in Y)$;
$7(Y) \leftrightarrow \forall_{\tau_1}\forall_{\tau_2}(\tau_1, \tau_2 \in \Phi \wedge \neg\sigma(\tau_2) \rightarrow \tau_1(\tau_2) \in Y)$;
$8(Y) \leftrightarrow \forall_\alpha(\alpha \in Y \rightarrow \text{`}\neg\alpha\text{'} \in Y)$;
$9(Y) \leftrightarrow \forall_\alpha\forall\beta(\alpha, \beta \in Y \rightarrow \text{`}\alpha \wedge \beta\text{'}, \text{`}\alpha \vee \beta\text{'}, \text{`}\alpha \rightarrow \beta\text{'}, \text{`}\alpha \leftrightarrow \beta\text{'} \in Y)$;
$10(Y) \leftrightarrow \forall_\alpha(\alpha \in Y \wedge \tau \in I \cup G \cup V \cup T \rightarrow \text{`}\forall_\tau\alpha\text{'}, \text{`}\exists_\tau\alpha\text{'} \in Y)$.

An outline of semantics

A Model, or a possible world is conceived in traditional logic as an ordered triple m = <S, P, R>, namely:

$$\text{A possible world } (\langle S, P, R \rangle)$$

where the first two elements in this triple are the universes of the dual-band model. And S is a set of the ontic subjects, which are universals of a specific category:

$$\text{Universal } (S) \wedge \exists_k[k \in \{r, c, s\} \wedge \forall x/S \text{ Category}/x\,(k)]$$

$$\text{Set/Attribute}^+ \text{ (P)} \wedge P \neq \emptyset, \text{ but the relation}$$
$$R \subseteq S \times P \wedge R \neq \emptyset \wedge s(R).$$

In the case – which is not excluded in advance – when $S = P$, the model is a single-band one.

Ontic subjects: species, genera and individuals are correlates of signs (names) and, as such, can never be either empty or contradictory, because one can not predicate anything meaningful about what is senseless.

"A name that does not mean anything is just no name at all ..."[5]

[5]See: [3].

However, the attributes are not so limited, because even ontic subjects can be reasonably judged – but only in negative sentences – that they are not anything that does not exist, that they are not entitled to attributes that are neither empty nor contradictory.

Aristotle gives an example of a "goat-stag" (An. Pr. I, 38. The first paragraph from the top), which – as a contrary product of fantasy – is no ontic subject, but it can be treated as an attribute, because it is true about every consistent object that it is not a goat-stag. However, any, even a negative statement about what exists in no sense (e.g. a goat-stag) does not make any sense.[6]

Schemes of traditional sentences, with the models: $\langle S, P, =\rangle$ take the meaning[7]:

$$\text{SaP} \leftrightarrow \forall x/S\, \exists y/P\, x = y, \text{ or } \text{SaP} \leftrightarrow P(\forall S),$$

where 'S' is in a simple supposition, unconditional and distributive. Similarly, you can save more diagrams:

$$\begin{aligned} \text{SeP} &\leftrightarrow \forall x/S\, \forall y/P\, x \neq y, \\ \text{SiP} &\leftrightarrow \exists x/S\, \exists y/P\, x = y, \\ \text{SoP} &\leftrightarrow \exists x/S\, \exists y/P\, x \neq y. \end{aligned}$$

Also additional patterns of William Hamilton can be reported similarly:

$$\begin{array}{ll} \forall x/S\, \forall y/P\, x = y & \exists x/S\, \forall y/P\, x = y, \\ \exists x/S\, \forall y/P\, x \neq y & \forall x/S\, \exists y P x \neq y. \end{array}$$

For any two-place situation (relation) R, we automatically receive 8 basic types of sentences:

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[6] Also in the case, if Aristotle had claimed that the goat-stag were no goat-stag, he would have only admitted that this anything did not exist, that it were nothing, and that sentences like SaP and SeP, in the sense of $\forall_x[S(x) \rightarrow P(x)]$ and $\forall_x[S(x) \rightarrow \neg P(x)]$ would be equivalent with respect to one another, for *empty S*. It is also possible that Aristotle, in some cases, confused *inherence* with *subsumption* and *ontological subject* with *subject of sentence*.

[7] Only one of several ways of interpreting the operators of Aristotle (a, e, i, o) made in modern classical predicate calculus – [4] in the definition of SaP: SaP $\leftrightarrow \{[\forall_x(Sx \rightarrow Px) \wedge \exists_x Sx \wedge \exists_x \neg Px] \vee \forall_x(Sx \leftrightarrow Px)\}$ – preserved the validity of all tautological modes of syllogistic traditional logic, without additional restrictions, although this interpretation is considered by the author as unnatural.

$$\forall x/S\,\exists y/P\, R/y(x)\,,\ \text{or } \mathrm{R}/\exists P\,(\forall S),$$
$$\forall x/S\,\forall y/P\,\neg R/y(x)\,,$$
$$\exists x/S\,\exists y/P\, R/y(x)\,,$$
$$\exists x/S\,\exists y/P\,\neg R/y(x)\,.$$
$$\forall x/S\,\forall y/P\, R/y(x)\,,$$
$$\exists x/S\,\forall y/P\, R/y(x)\,,$$
$$\exists x/S\,\forall y/P\,\neg R/y(x)\,,$$
$$\forall x/S\,\exists y/P\,\neg Ry(x).$$

We accept the following abbreviations:

$\Diamond$ =: formally possible

$\Box$ =: formally necessary

Then we define:

$$\Diamonddot Ag \leftrightarrow \neg\boxdot\neg \mathrm{A}(g), \text{ at least sometimes it is present (is actual)} \tag{1}$$
$$\neg\Diamonddot Ax \leftrightarrow \boxdot\neg \mathrm{A}(x), \text{ even sometimes it is not present} \tag{2}$$
$$\neg\Diamonddot\neg \mathrm{A}(x) \leftrightarrow \boxdot \mathrm{A}(x), \text{ it is not even sometimes non-present} \tag{3}$$
$$\Diamonddot\neg \mathrm{A}(x) \leftrightarrow \neg\boxdot \mathrm{A}(x), \text{ at least sometimes it is non- present} \tag{4}$$
$$-y(x) \leftrightarrow \neg(y(x)), \text{ for } `x', `y' \in \Phi \tag{5}$$
$$\Diamond(x) \leftrightarrow -\sigma(x) \leftrightarrow \neg(\sigma(x)) \leftrightarrow \neg\Box(\neg x) \tag{6}$$
$$\Box(x) \leftrightarrow \sigma(\neg x) \leftrightarrow \neg(-\sigma(\neg x)) \leftrightarrow \neg\Diamond(\neg x) \tag{7}$$
$$\Box(x) \leftrightarrow \sigma(x) \leftrightarrow \neg(-\sigma(x)) \leftrightarrow \neg\Diamond(x) \tag{8}$$
$$\Diamond(\neg x) \leftrightarrow -\sigma(\neg x) \leftrightarrow \neg(\sigma(\neg x)) \leftrightarrow \neg\Box(x) \tag{9}$$
$$\sigma(\neg x) \rightarrow -\sigma(x) \tag{10}$$
$$\sigma(x) \leftrightarrow \Box(\neg x) \rightarrow \Diamond(\neg x) \leftrightarrow -\sigma(\neg x) \tag{11}$$

Accepted axioms of the system:

$$-\sigma(x) \leftrightarrow x(x), \neg\sigma(g) \leftrightarrow \exists_x g(x) \tag{Axiom 0}$$
$$\sigma(\neg\boxdot \mathrm{A}(x)) \rightarrow A(x), \text{ or } \Box(\boxdot \mathrm{A}(x)) \rightarrow A(x) \tag{Axiom 1}$$
$$\sigma(\neg(f \rightarrow g)) \rightarrow [\sigma(\neg f) \rightarrow \sigma(\neg g)], \text{ or } \Box(f \rightarrow g) \rightarrow (\Box(f) \rightarrow \Box(g)) \tag{Axiom 2}$$

Now take the following substitutions in the Axiom 2: $\mathrm{f}|\Diamonddot\neg\mathrm{A}(\mathrm{f}), \mathrm{g}|\Diamonddot\neg\mathrm{A}(\mathrm{g})$

$$\sigma(\neg(\Diamond\neg\mathrm{A}(f) \to \Diamond\neg\mathrm{A}(g))) \to (\sigma(\neg(\Diamond\neg\mathrm{A}(f))) \to \sigma(\Diamond\neg\mathrm{A}(g))), \text{or}$$
$$\Box(\Diamond\neg\mathrm{A}(f) \to \Diamond\neg\mathrm{A}(g)) \to (\Box(\Diamond\neg\mathrm{A}(f)) \to \Box(\Diamond\neg\mathrm{A}(g)))$$
(Axiom 2')

Let us make the following substitutions in the Axiom 2: $\mathrm{f}|\Diamond\mathrm{Af}, \mathrm{g}|\Diamond\mathrm{Ag}$

$$\sigma(\neg(\Diamond Af) \to \Diamond Ag) \to (\sigma(\neg(\Diamond Af)) \to \sigma(\neg(\Diamond Ag))), \text{ or}$$
$$\Box(\Diamond Af \to \Diamond Ag) \to (\Box(\Diamond Af) \to \Box(\Diamond Ag))$$
(Axiom 2")

For $(\alpha, \beta) \in \Psi, \gamma^k =: \gamma \leftrightarrow$ Category/$\gamma(k)$, $\quad g \in I \cup G \cup T \cup V$

(Rules of inference)

$\vdash\alpha[g^k]$, so $\vdash\alpha[\mathrm{g^k}|\gamma^\mathrm{k}]$ (Rule of substitution)

$\vdash(\alpha \to \beta), \vdash\alpha$, so $\vdash\beta$ (Rule of detachment)

$\vdash\alpha$, so $\vdash\sigma(-\alpha)$ (Gödel's rule)

Let

$$M = \{m : m = \langle S, P, R\rangle \wedge \mathrm{Model}(\langle S, P, R\rangle)\}$$

Then:

$$\text{Set/The possible World}^+(M) \tag{12}$$

Let in turn

$$\mathbf{m} = \mathbf{m}_t \leftrightarrow S/\mathbf{m} = \{x : A_t(x)\}, \quad \text{for } t \in T. \tag{13}$$

Then: The actual World/$t(m_t)$, for $t \in T$:

$$M_t = \{\mathbf{m}_t : t \in T\} \to \text{Set/actual World}^+/t(M_t) \tag{14}$$

Finally, let $m = m_\Diamond \leftrightarrow S/m \subseteq \{x : \Diamond Ax\}$. Then:
Real World ($\mathbf{m}_\Diamond$) and

$$M_\Diamond = \{M_t : t \in T\} \to \text{Set/Real World}^+(M_\Diamond) \tag{15}$$

As we can easily see, the formal modalities are defined in all classes of models, and temporal modalities and real ones – only in the class of real models: $M_\Diamond$.

In this case, also the notions of *raison d'être* and *sufficient reason for being* that interest us – as ones regarding the real reality – must be considered in relation to the class of real worlds $M_\Diamond$.

Let us denote the *ratio* of a *raison d'être* with ϱ, and the ratio of sufficient reason with δ.

Then we propose – as definitions for reporting conventions of meaning of these concepts in classical philosophy – the following equivalences:

$$\varrho/y(x) \leftrightarrow \neg\Diamond(\Diamond Ay \wedge \boxdot\neg \mathrm{A}(y)), \text{whereas}^8 \qquad \text{(raison d'etre)}$$
$$\delta/y(x) \leftrightarrow \varrho/y(x) \wedge \forall_z(\varrho/x(z) \rightarrow x = z) \qquad \text{(suff. reason for being)}$$

The pair $\langle M_\Diamond \Gamma\rangle$, where first element is the class real worlds $M_\Diamond$ and the second one – the Accessibility Relation Γ, will be called frame or, better, Rational World

The Accessibility Relation $\Gamma/n(m)$ – read: the real reason for the existence of the world n is the real existence of the world m – is defined as follows:

$$\Gamma/n(m) \leftrightarrow \forall y/S/n \ \ \exists x/S/m \ \varrho/y(x)$$
$$\Gamma/n(m) \leftrightarrow \varrho/\forall S/n \ (\exists S/m\,)$$

From the real world **n**, the real world **m** is accessible $\leftrightarrow$ for each subject of the world **n**, there is a reason of its being in the subjects of the world **m**:

Theorem 1. *Reflexive*$/M_\Diamond(\Gamma)$

Proof: $\forall \mathbf{m}/M_\Diamond\Gamma/\mathbf{m}(\mathbf{m})$,
because $\forall x/S/m/M_\Diamond\varrho/x(x)$,
because $\forall x/S/m/M_\Diamond\neg\Diamond(\Diamond Ax \wedge \neg\Diamond Ax)$
because $\forall x/S/m/M_\Diamond\sigma(\Diamond Ax \wedge \neg\Diamond Ax)$,
because $\vdash\alpha \rightarrow \alpha$, RG

Theorem 2. *Serial*$/M_\Diamond(\Gamma)$ *or* $\forall \mathbf{n}/M_\Diamond\exists \mathbf{m}/M_\Diamond\Gamma/\mathbf{n}(\mathbf{m})$

---

[8]So $\varrho/g(f) \leftrightarrow \sigma(\Diamond Ag \wedge \boxdot\neg \mathrm{A}(f)$, or $\varrho/g(f) \leftrightarrow \Box(\Diamond Ag \rightarrow \Diamond Af)$. It is not possible (because it is contrary) that the sequence is a real being, and its reason – non-existence. The absence within the being (*privatio*) also exists. The lack is a non-entity only if it is inconsistent, i.e. absent and present in the same respect and in the same case (in the same place, at the same time).

PROOF: because Theorem 1.

THEOREM 3. *Transitive*/$M_{\Diamond}(\Gamma)$ *or* $\Gamma/\mathbf{n}(\mathbf{l}) \wedge \Gamma/\mathbf{l}(\mathbf{m}) \rightarrow \Gamma/\mathbf{n}(\mathbf{m})$
for $\forall(\mathbf{m}, \mathbf{n}, \mathbf{l}) \in M_{\Diamond}$

PROOF:

1. $\Gamma/\mathbf{n}(\mathbf{l})$, assumption
2. $\Gamma/\mathbf{l}(\mathbf{m})$, assumption
3. $\forall y/\mathbf{n}\, \exists x/\mathbf{l}\; \varrho/y(x)$, because 1.
4. $\forall y/\mathbf{l}\, \exists x/\mathbf{m}\; \varrho/y(x)$, because 2.
5. $S/\mathbf{n}(y)$, because 3.
6. $\exists x/\mathbf{m}\; \varrho/y(x)$, because 4.
7. $\forall y/n\, \exists x/m\; \varrho/y(x)$, because 5 and 6
8. $\Gamma/\mathbf{n}(\mathbf{m})$, because 7.

THEOREM 4. *Quasi-ordering*$M_{\Diamond}/(\Gamma)$

PROOF: because Reflexive/$M_{\Diamond}(\Gamma)$ and Transitive/$M_{\Diamond}(\Gamma)$

THEOREM 5. $\neg$*Symmetric*/$M_{\Diamond}(\Gamma) \wedge \neg$*Antisymmetric*/$M_{\Diamond}(\Gamma) \wedge$
$\wedge \neg$*Connective*/$M_{\Diamond}(\Gamma) \wedge \neg$*Euclidean*/$M_{\Diamond}(\Gamma)$

As presented here, the Rational World $\langle M_{\Diamond}, \Gamma \rangle$ corresponds to the system `KD4`.

Interpretation of the language in the semantic models

All correct language expressions are the signs and have correlates in the possible and real worlds. Signs are listed in material supposition, in the form of a double quote or quasi-quote, and their correlates – in simple or formal supposition.

Thus we denote each variable and nominal or propositional function by the use of quasi-quotes, such as ‘x’, ‘$-x$’, ‘$y(x)$’, ‘$\sigma(y(x) \wedge \neg y(x)$’, constants – by the use of ordinary quotes, e.g. “$\neg$”, “$-$”, “r”, “c”, “s”, “σ”, and their correlates – without any quotes, for example, x,$-x$, $y(x)$, $\sigma(y(x) \wedge \neg y(x)$, $\neg$, $-$, r, c, s, σ.

The value of the expression is a positive correlate: 1 – a non-contradictory projection in possible worlds or a fact in the real worlds (F), and a negative correlate: 0 – an inconsistent projection, or an anti-fact.

$$F(x) \leftrightarrow s(x) \wedge A(x), \qquad x \in F/m_t \leftrightarrow F(x) \wedge x \in S/m_t\,.$$

The definition of the positive values of expressions:

If we use the metalinguistic letter α – taken in material supposition – to denote any correct expression of the language, then we get a negative value directly from the negation of the positive:

$$\begin{aligned}
\mathrm{W}\,(\alpha, m) &= 0 \leftrightarrow \mathrm{W}\,(\alpha, m) \neq 1\\
\mathrm{W}\,(`x\text{'}, m) &= 1 \leftrightarrow m \in M \wedge x \in S/m\,;\\
\mathrm{W}\,(`\Box(x)\text{'}, m) &= 1 \leftrightarrow m \in M \wedge \sigma(-x) \wedge x \in S/m\,;\\
\mathrm{W}\,(`\Diamond(x)\text{'}, m) &= 1 \leftrightarrow m \in M \wedge \neg\sigma(x) \wedge x \in S/m\,;\\
\mathrm{W}\,(`x\text{'}, m_t) &= 1 \leftrightarrow t \in T \wedge x \in S/m_t\,;\\
\mathrm{W}\,(`x\text{'}, m_t) &= 0 \leftrightarrow \mathrm{W}\,(x, m_t) \neq 1 \leftrightarrow t \notin T \vee x \notin S/m_t \leftrightarrow\\
&\qquad\qquad \leftrightarrow t \notin T \vee x \notin S/m_t \vee A_t(x);\\
\mathrm{W}\,(`-x\text{'}, t) &= 1 \leftrightarrow \mathrm{W}\,(x, m_t) = 0;\\
\mathrm{W}\,(`x \wedge y\text{'}, t) &= 1 \leftrightarrow \mathrm{W}\,(x, m_t) = 1 \wedge \mathrm{W}\,(y, m_t) = 1;\\
\mathrm{W}\,(`\forall xy\text{'}, m_t) &= 1 \leftrightarrow \forall x/S/m_t F(y);\\
\mathrm{W}\,(`\exists xy\text{'}, m_t) &= 1 \leftrightarrow \exists x/S/m_t F(y);\\
\mathrm{W}\,(`A_t(x)\text{'}, m_t) &= 1 \leftrightarrow x \in S/m_t \quad \text{for } t \in T;\\
\mathrm{W}\,(`A(x)\text{'}, m_t) &= 1 \leftrightarrow \mathrm{W}\,(A_t(x), m_t) = 1;\\
\mathrm{W}\,(`\Diamond Ax\text{'}, m_\Diamond) &= 1 \leftrightarrow m_\Diamond \in M_\Diamond \wedge \exists t/T\,(A_t(x) \wedge x \in S/m_\Diamond);\\
\mathrm{W}\,(`\boxdot \mathrm{A}(x)\text{'}, m_\Diamond) &= 1 \leftrightarrow m_\Diamond \in M_\Diamond \wedge \forall t/T\,(m_t \in M_\Diamond \wedge x \in S/m_t\,);\\
\forall \alpha/\Phi \cup \Psi\,((\mathrm{W}\,(\alpha, m) &= 1 \vee \mathrm{W}\,(\alpha, m_t) = 1) \rightarrow \mathrm{W}\,(\alpha, m_\Diamond) = 1).
\end{aligned}$$

Let '$\underline{\Box}(x)$' be a real necessity, and '$\underline{\Diamond}(x)$' – a real possibility.

$$\begin{aligned}
&\mathrm{W}\,(`\underline{\Box}(x)\text{'}, m) = 1 \leftrightarrow m \in M_\Diamond \wedge \mathrm{W}\,(`A(x)\text{'}, m) =\\
&\qquad\qquad = 1 \wedge \forall n/M_\Diamond(\Gamma/m(n) \rightarrow \mathrm{W}\,(`A(x)\text{'}, n) = 1);\\
&\mathrm{W}\,\big(`\underline{\Diamond}(x)\text{'}, m\big) = 1 \leftrightarrow m \in M_\Diamond \wedge \mathrm{W}\,(`A(x)\text{'}, m) =\\
&\qquad\qquad = 1 \wedge \exists n/M_\Diamond(\Gamma/m(n) \rightarrow \mathrm{W}\,(`A(x)\text{'}, n) = 1);
\end{aligned}$$

$$W(\alpha, M) = 1 \leftrightarrow \forall m/M\, W(\alpha, m) = 1;$$
$$W(\alpha, M_t) = 1 \leftrightarrow \forall m/M_t\, W(\alpha, m) = 1, \quad \text{for } t \in T;$$
$$W(\alpha, M_\Diamond) = 1 \leftrightarrow \forall m/M_\Diamond W(\alpha, m) = 1.$$

The formalization of theodicy

As already said, the sign ϱ denotes the ratio of the reason of being, and δ – the ratio of the sufficient reason of being.

And the following definitions were adopted:

$$\varrho/y(x) \leftrightarrow \neg\Diamond(\Diamond Ay \wedge \boxdot\neg A(x)),$$
$$\delta/y(x) \leftrightarrow \varrho/y(x) \wedge \forall_z(\varrho/x(z) \rightarrow x = z).$$

Then the principle of raison d'être: $\forall_y \exists_x x\varrho y$ is a certainty (because ϱ is a reflexive relation), and the principle of the sufficient reason for being: $\forall_y \exists_x x\delta y$ is a *postulate*, in the traditional sense of the words.

In the *Summa Theologica* by St. Thomas Aquinas (vol. I q.2 a.3), there are two articles that create a lot of trouble for interpreters:

> But this cannot go on to infinity, because then there would be no first mover, and, consequently, no other mover; seeing that subsequent movers move only in as much as they are put in motion by the first mover.
> Now in efficient causes it is not possible to go on to infinity, because in all efficient causes following in order, the first is the cause of the intermediate cause, and the intermediate is the cause of the ultimate cause, whether the intermediate cause be several, or only one. Now to take away the cause is to take away the effect. Therefore, if there be no first cause among efficient causes, there will be no ultimate, nor any intermediate cause. But if in efficient causes it is possible to go on to infinity, there will be no first efficient cause, neither will there be an ultimate effect, nor any intermediate efficient causes; all of which is plainly false.[9]

Using the concepts of raison d'être, intuitions and intentions contained in this text can be translated into symbolic language adopted above.

[9]http://www.newadvent.org/summa/1002.htm Home > Summa Theologica > First Part > Question 2. The existence of God

1. $b(x) \leftrightarrow \forall_y(\Diamonddot Ay \rightarrow \varrho/y(x)\,)$ The definition of the term "b" ("God")
2. $b(b) \leftrightarrow \forall_y(\Diamonddot Ay \rightarrow \varrho/y(b)\,)$ [1, x|b]
3. $b(b), [\neg\sigma(b), \Diamond(b)]$ [Axiom 0]
4. $\forall_y[\Diamonddot Ay \rightarrow \varrho/y(b)\,]$ [2,3]
5. $\varrho/y(b) \leftrightarrow \Box(\Diamonddot Ay \rightarrow \Diamonddot Ab)$ [Df. ϱ]
6. $\forall_y[\Diamonddot Ay \rightarrow \Box(\Diamonddot Ay \rightarrow \Diamonddot Ab)]$ [4,5]
7. $\Diamonddot Ay \rightarrow \Box(\Diamonddot Ay \rightarrow \Diamonddot Ab)$ [O$\forall$: 6]
8. $\Diamonddot Ay \rightarrow (\Diamonddot Ay \rightarrow \Diamonddot Ab)$ [7, $\Box\alpha \rightarrow \alpha$]
9. $(\Diamonddot Ay \rightarrow \Diamonddot Ab)$ [8]
10. $\neg\Diamonddot Ab \rightarrow \neg\Diamonddot Ay$ [9]
11. $\boxdot\neg\mathrm{A}(b) \rightarrow \boxdot\neg\mathrm{A}(y)$ [10, Df. $\neg\Diamonddot Ax \leftrightarrow \boxdot\neg\mathrm{A}(x)$]
12. $\forall_y(\boxdot\neg\mathrm{A}(b) \rightarrow \boxdot\neg\mathrm{A}(y))$ [D$\forall$: 11]
13. $\boxdot\neg\mathrm{A}(b) \rightarrow \forall_y\boxdot\neg\mathrm{A}(y)$ [12]
14. $\neg\forall_y\boxdot\neg\mathrm{A}(y)$ [it is not true that nothing exists]
15. $\neg\boxdot\neg\mathrm{A}(b)$ [it is not true that God does not exist], [13, 14]

The cited evidence is problematic for the adoption of line 3, namely the idea $\neg\sigma(b)$. Since in the definition in row 1 the term "b" is the predicate, it must be shown that: $\neg\sigma(\forall_y[\Diamonddot Ay \rightarrow \varrho/y(x)\,])$, which is based on Axiom 0: $\exists_x\forall_y[\Diamonddot Ay \rightarrow \varrho/y(x)\,]$. Let us assume by contradiction that it is not:

1. $\neg\exists_x\forall_y[\Diamonddot Ay \rightarrow \varrho/y(x)\,]$ a.p.c. assumption of proof by contradiction
2. $\forall_x\exists_y[\Diamonddot Ay \wedge \neg\varrho/y(x)\,]$, [1]
3. $\exists_y[\Diamonddot Ay \wedge \neg\varrho/y(x)\,]$ [O$\forall$: 2]
4. $\Diamonddot Aa_x \wedge \neg\varrho/a_x(x)$ [O$\exists$: 3]
5. $\neg\varrho/a_x(x)$ [O$\exists$: 3]
6. $\Diamond(\Diamonddot Aa_x \wedge \neg\Diamonddot Ax)$ [Df. ϱ: 5]
7. $\Diamond\boxdot\neg\mathrm{A}(x)$ [6]

Consclusions

Based on reflection herein, we can draw a number of surprising conclusions about differences in traditional and contemporary understanding of reality in the aspect of suitability for logic:

1. Aristotle and the followers of his philosophy introduced a system of several ontic categories, which can be reduced to the categories of

things, *features* and *situations*.[10] The elements of each category, their individuals, can be used either as *ontological subjects* or *attributes*, but the subjects can never be contradictory. All of these can be seen in various aspects, which are indeed the *properties* of category items, but they do not belong to a category themselves. The *essence* and *existence* are the most important aspects of the category. Hence, the *possibility* – understood as *consistency* in the philosophical tradition – and consequently the *necessity* are twofold: of the essence – formal and of the existence – real.

2. Each *sign* of the language has an extra-linguistic *correlate*. The inscription or the sound that has no correlate is no sign at all. The formal possibility (necessity) confirms that it is possible (with the necessity is so) that the bottom line is as it was presented; while the real modalities declare the possibility (necessity) of the real existence of the situation, the reality declares the possibility of the fact. Therefore, the classical syntax can not be pure morphology of the language. To note in the syntax that the inscription – e.g. of the type '$y(x)$' or '$g(x)$' – is a correct expression, it must be assumed that the ontic subject x is consistent: $\neg\sigma(x) \to$ ('$y(x)$', '$g(x)$' $\in$ set of propositional formulas).
3. The fundamental difference between ancient and modern semantics is the diverse understanding of general categorical sentences. Today, a sentence of the type SaP does not say anything about the subject S, but about all the objects of the universe of speech that if they have the attribute S, they are also entitled to attribute P. Contemporary conditional quantification over all individuals is different from the traditional unconditional quantification and is also not reducible to the so-called quantification of limited scope, because the latter is just another recording of the conditional quantification. Ever since Aristotle created syllogistic, the formula SaP has unconditionally meant that every particular in the field of S is a subject of the attribute P. In addition, any statement with the empty or inconsistent subject is meaningless.
4. The concepts of essence and existence of the correlate of a sign are perhaps the most important for the cause of understanding the se-

[10] The set of all universals breaks up into separate families of species being the main ideals of subsumption generated by particular items of the categories.

mantic models. Because if a correlate is the result of cognitive projection of the sign into some extralinguistic reality, and as for classic thinkers – professing usually a moderate conceptual realism – correlates were the codes *cum fundamento in re*, today – probably under the influence of the Kant's gnoseology – we often meet the agnostic view that correlates are just pure projection in a remote or even completely unknown reality. The common belief that the constant is at most only the change of the world, because everything changes, everything comes into being continuously, raises a difficult issue of solving the problem of identification of individuals in their volatility. And in this case, probably Aristotle was the first to discover the way out of this aporia, escape in the direction of invariance just through the essence of the changeable individuals of each category.

5. In our times, the logical analysis of the arguments for the existence of the absolute becomes a trend quite common in the so-called philosophical logic. And two opposed cultures of the logical and philosophical analysis collided, or perhaps even crashed on this ground.

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[2] Aristotle, Prior Analytics, http://ebooks.adelaide.edu.au/a/aristotle/a8pra/index.html. Translated by A. J. Jenkinson.

[3] I. Dąmbska, *W sprawie tzw. nazw pustych*, **Przegląd Filozoficzny** 1-3/XLIV (1948), pp. 77–81.

[4] S. Jaśkowski, *O interpretacji zdań kategorycznych Arystotelesa w rachunku predykatów*, **Studia Societatis Scientiarum Torunensis** 2/3 (1950).

[5] E. Nieznański, *A logical analysis of philosophical arguments based on Leibniz's principle of the sufficient reason of existence*, **Bulletin of the Section of Logic** 12 (1983), pp. 188–193.

[6] E. Nieznański, *Logical Analysis of Thomism*, [in:] Jan Srzednicki (ed.) **Initiatives in Logic**, volume 2 of **Reason and Argument** Springer Netherlands, 1987, pp. 128–155.

[7] E. Nieznański, *Elements of modal theodicy*, **Bulletin of the Section of Logic** 37:3/4 (2008), pp. 253–264.

[8] E. Nieznański, *Concepts of Proof and Formalized Arguments ex gradibus perfectionis*, [in:] Mirosław Szatkowski (ed.) **Ontological Proofs Today**, chapter 18, De Gruyter, Berlin, Boston, 2012, pp. 379–390.
[9] E. Nieznański, *The first formalized proof of the indestructibility of a subsistent form*, **Studies in East European Thought** 65/1-2 (2013), pp. 65–73.
[10] E. Nieznański, *Towards a Formalization of Thomistic Theodicy. Formalized Attempts to Set Formal-Logical Bases to State First Elements of Relations Considered in the Thomistic Theodicy*, volume 5 of **European Studies in Theology, Philosophy and History of Religions**, Peter Lang Edition, Frankfurt am Main, Berlin, Bern, Bruxelles, New York, Oxford, Wien,2013.

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Appendix

In this Appendix will be presented – in the computer program *Prover9* – two different proofs (**L** and **G**) for the existence and unity of God. **L** will be moreover in two versions: non-modal **L1** and **L2**-modal.

L. The inspired by Gottfried Wilhelm Leibniz[1] evidence for the existence and unity of God, based on the principle of sufficient reason for being

<u>L1. non-modal version</u>

Explanation extra-logical constants:

[1]See (Theodicy§7: G VI 10607/H 12728)
[G] Die philosophischen Schriften. 7 vols. Edited by C. I. Gerhardt. Berlin, 187590. Reprint, Hildesheim: Georg Olms, 1965.
– Political Writings. Edited by Patrick Riley. Second edition. Cambridge: Cambridge University Press, 1988.
[H] Theodicy: Essays on the Goodness of God, the Freedom on Man and the Origin of Evil. Translated by E. M. Huggard. La Salle, IL: Open Court, 1985.

$T(x) = x$ is actual at least sometimes (temporally contingent being);
$R(x, y) =:$ the existence of x is the reason for the existence of y;
$S(x, y) =:$ the existence of x is a sufficient reason for the existence of y;
$G(x) = x$ is God.

L1a. God exists

assign(report_stderr, 2).
set(ignore_option_dependencies). % GUI handles dependencies

if(Prover9). % Options for Prover9
    assign(max_seconds, 60).
end_if.

if(Mace4). % Options for Mace4
    assign(max_seconds, 60).
end_if.

formulas(assumptions).

$R(x, y) < - > (T(y) - > T(x))$.
$S(x, y) < - >$ all $z(R(z, x) - > z = x) \& R(x, y)$.
$T(y) - >$ exists $x S(x, y)$.
$G(x) < - >$ all $y(T(y) - > S(x, y))$.

end_of_list.

formulas(goals).

exists $x G(x)$.

end_of_list.

L1b. God is the only one

assign(report_stderr, 2).
set(ignore_option_dependencies). % GUI handles dependencies

if(Prover9). % Options for Prover9
    assign(max_seconds, 60).
end_if.

if(Mace4). % Options for Mace4
assign(max_seconds, 60).
end_if.

formulas(assumptions).

$R(x,y) <-> (T(y)-> T(x))$.
$S(x,y) <-> \text{all } z(R(z,x)-> z=x)\&R(x,y)$.
$T(y)->\text{exists } xS(x,y)$.
$G(x) <-> \text{all } y(T(y)-> S(x,y))$.
exists $yT(y)$.

end_of_list.

formulas(goals).

$G(x)\&G(y)-> x=y$.

end_of_list.

L2 modall version

Explanation extra-logical constants:
$E(x)$ =: x (real) exists;
$C(x)$ =: x iscontradictory;
$N(x)$ =: x is necessary;
$R(x,y)$ =: the existence of x is the reason for the existence of y;
$S(x,y)$ =: the existence of x is a sufficient reason for the existence of y;
$G(x) = x$ is God.

L2a. God exists

set(ignore_option_dependencies). % GUI handles dependencies

if(Prover9). % Options for Prover9
assign(max_seconds, 60).
end_if.

if(Mace4). % Options for Mace4
assign(max_seconds, 60).
end_if.

formulas(assumptions).

$C(x) < - > (E(x)\& - E(x))$.
$N(x) < - > (-E(x)- > C(x))$.
$R(x, y) < - > (E(y)- > N(x))$.
$S(x, y) < - > R(x, y)\&$ all $z(R(z, x)- > z = x)$.
$G(x) < - >$ all $y(E(y)- > S(x, y))$.
all y exists $xS(x, y)$.

end_of_list.

formulas(goals).

exists $xG(x)$.
end_of_list.

L2b. God is the only one

assign(report_stderr, 2).
set(ignore_option_dependencies). % GUI handles dependencies

if(Prover9). % Options for Prover9
    assign(max_seconds, 60).
end_if.

if(Mace4). % Options for Mace4
    assign(max_seconds, 60).
end_if.

if(Prover9). % Additional input for Prover9
assign(report_stderr, 2).
set(ignore_option_dependencies).
end_if.

formulas(assumptions).

$C(x) < - > (E(x)\& - E(x))$.
$N(x) < - > (-E(x)- > C(x))$.
$R(x, y) < - > (E(y)- > N(x))$.
$S(x, y) < - > R(x, y)\&$ all $z(R(z, x)- > z = x)$.
$G(x) < - >$ all $y(E(y)- > S(x, y))$.

all y exists $xS(x,y)$.

end_of_list.

formulas(goals).

$G(x)\&G(y)->x=y$.

end_of_list.

G. The inspired by Kurt Gödel proof for the Gods existence and unity[2], based on the concept of positivity in the highest degree

Explanation extra-logical constants:

$C(x,y)$ =: the concept of x is included in the concept of of y (inclusion of concepts);
$J(x,y)$ =: x is y;
0 =: empty concept; $C(x,0) \leftrightarrow \forall y C(x,y)$.
$U(x) = x$ is at most one;
$P(x) = x$ is positive in the highest degree;
$G(x) = x$ is God.

G1a. God exists

assign(report_stderr, 2).
set(ignore_option_dependencies). % GUI handles dependencies

if(Prover9). % Options for Prover9
assign(max_seconds, 60).
end_if.

if(Mace4). % Options for Mace4
assign(max_seconds, 60).
end_if.

formulas(assumptions).

$P(x) <-> (\text{all } y(P(y)-> J(x,y)))\& (\text{all } z \text{ all } y(P(z)\&J(z,y)-> P(y)))$.

[2]See Gödel K. (1995), Collected Works, ed. S. Feferman, Oxford Univ. Press, "Ontological proof", in: Collected Works, vol. 3, 403–404.

$U(x) <-> (\text{all } y(J(y,x)-> J(x,y)))$.
$C(x,y) <-> (\text{all } z(C(z,x)-> C(z,y)))$.
$J(x,y) <-> -C(x,0)\&C(x,y)$.
$G(x) <-> P(x)\&U(x)$.

end_of_list.

formulas(goals).

exists x G(x).

end_of_list.

G1b. God is the only one

========== INPUT ==========

assign(report_stderr,2).
set(ignore_option_dependencies).
if(Prover9).
% Conditional input included.
assign(max_seconds,60).
end_if.
if(Mace4).
% Conditional input omitted.
end_if.

formulas(assumptions).
$P(x) <-> (\text{all } y(P(y)-> J(x,y)))\& (\text{all } z \text{ all } y(P(z)\&J(z,y)-> P(y)))$.
$x = y <-> J(x,y)\&J(y,x)$.
$G(x) <-> P(x)$.
end_of_list.

formulas(goals).
$G(x)\&G(y)-> x = y$.
end_of_list.

========== end of input ==========