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SEMANTICS FOR SŁUPECKI'S SYSTEM OF SYLLOGISTIC

Abstract

By modifying P. Kulicki's, a sound and complete semantics for Słupecki's system of Syllogistic is presented. The proof is independent of axiomatic rejection.

Kulicki [1], among other things, introduced a semantics for the system of Syllogistic presented by Słupecki, which is a minimal system including all the laws of Aristotle.

In this note, a sound and complete semantics for the system is introduced by modifying Kulicki's. Comment on a fault in his semantics is also given. A characteristic of our proof is that it is independent of axiomatic rejection.

For details of Słupecki's work, see Kulicki [1].

1. Syntax, semantics and soundness

Atomic formulas of Słupecki's system are $\mathcal{X}a\mathcal{Y}$ and $\mathcal{X}i\mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ are name variables. *Compound formulas* are constructed from atomic ones by using the propositional connectives $\neg$ (negation), $\wedge$ (conjunction), $\rightarrow$ (implication), and so on. Name variables and formulas will be denoted by $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{U}, \mathcal{V}, \ldots$ and $\alpha, \beta, \varphi, \psi, \ldots$ (possibly with subscripts), respectively.

Słupecki's system **S** is the one that is based on the classical propositional calculus complemented by the following four axioms schemes (S1)–(S4):

(S1) $\mathcal{X}a\mathcal{Y} \rightarrow \mathcal{X}i\mathcal{Y}$,

(S2) $\mathcal{X}i\mathcal{Y} \rightarrow \mathcal{Y}i\mathcal{X}$,

(S3) $\mathcal{X}a\mathcal{Y} \wedge \mathcal{Y}a\mathcal{Z} \to \mathcal{X}a\mathcal{Z}$,
(S4) $\mathcal{X}i\mathcal{Y} \wedge \mathcal{Y}a\mathcal{Z} \to \mathcal{X}i\mathcal{Z}$.

Our *model* for **S** is the triplet $\langle f, \mathcal{D}^a, \mathcal{D}^i \rangle$, where f is a function which correlates each name variable with a non-empty set, while $\mathcal{D}^a$ and $\mathcal{D}^i$ are sets (of sets) such that $\mathcal{D}^a \subseteq \mathcal{D}^i$.

Given a model $\mathcal{M} = \langle f, \mathcal{D}^a, \mathcal{D}^i \rangle$, the notion of *truth* in $\mathcal{M}$ of a formula α (notation: $\mathcal{M} \models \alpha$) is defined inductively as follows:

$$\begin{aligned}
\mathcal{M} \models \mathcal{X}a\mathcal{Y} &\iff f\mathcal{X} \subseteq f\mathcal{Y} \textit{ and } f\mathcal{Y} \in \mathcal{D}^i \\
&\qquad \textit{and } (f\mathcal{X} = f\mathcal{Y} \textit{ implies } f\mathcal{Y} \in \mathcal{D}^a), \\
\mathcal{M} \models \mathcal{X}i\mathcal{Y} &\iff f\mathcal{X} \cap f\mathcal{Y} \neq \emptyset \\
&\qquad \textit{and } (f\mathcal{X} = f\mathcal{Y} \textit{ implies } f\mathcal{Y} \in \mathcal{D}^i), \\
\mathcal{M} \models \neg\alpha &\iff \mathcal{M} \not\models \alpha, \\
\mathcal{M} \models \alpha \wedge \beta &\iff \mathcal{M} \models \alpha \textit{ and } \mathcal{M} \models \beta, \\
\mathcal{M} \models \alpha \to \beta &\iff \mathcal{M} \not\models \alpha \textit{ or } \mathcal{M} \models \beta,
\end{aligned}$$

and so on.

It is to be remarked that, $\mathcal{M} \models \mathcal{X}a\mathcal{X}$ iff $f\mathcal{X} \in \mathcal{D}^a$, while $\mathcal{M} \models \mathcal{X}i\mathcal{X}$ iff $f\mathcal{X} \in \mathcal{D}^i$, since $\mathcal{D}^a \subseteq \mathcal{D}^i$ and $f\mathcal{X} \neq \emptyset$.

THEOREM 1.1 (Soundness). *If a formula is provable in* **S***, it is true in every model.*

PROOF: It suffices to check the axioms (S1)–(S4). Let $\mathcal{M} = \langle f, \mathcal{D}^a, \mathcal{D}^i \rangle$ be an arbitrary model.

(S1) Suppose $\mathcal{M} \models \mathcal{X}a\mathcal{Y}$. Then, $f\mathcal{X} \cap f\mathcal{Y} = f\mathcal{X} \neq \emptyset$; $f\mathcal{Y} \in \mathcal{D}^i$ whether $f\mathcal{X} = f\mathcal{Y}$ or not; hence $\mathcal{M} \models \mathcal{X}i\mathcal{Y}$.

(S2) $\mathcal{M} \models \mathcal{X}i\mathcal{Y}$ immediately implies $\mathcal{M} \models \mathcal{Y}i\mathcal{X}$.

(S3) Suppose $\mathcal{M} \models \mathcal{X}a\mathcal{Y} \wedge \mathcal{Y}a\mathcal{Z}$. Then, $f\mathcal{X} \subseteq f\mathcal{Y} \subseteq f\mathcal{Z}$; $f\mathcal{Z} \in \mathcal{D}^i$; if $f\mathcal{X} = f\mathcal{Z}$, then $f\mathcal{X} = f\mathcal{Y} = f\mathcal{Z}$ and so $f\mathcal{Z} \in \mathcal{D}^a$; hence $\mathcal{M} \models \mathcal{X}a\mathcal{Z}$.

(S4) Suppose $\mathcal{M} \models \mathcal{X}i\mathcal{Y} \wedge \mathcal{Y}a\mathcal{Z}$. Then, $f\mathcal{X} \cap f\mathcal{Z} \supseteq f\mathcal{X} \cap f\mathcal{Y} \neq \emptyset$; $f\mathcal{Z} \in \mathcal{D}^i$ whether $f\mathcal{X} = f\mathcal{Z}$ or not; hence $\mathcal{M} \models \mathcal{X}i\mathcal{Z}$. □

The following example is crucial in our pointing out the incompleteness of **S** with respect to Kulicki's semantics (cf. Examples 3.1 and 3.2 below).

Example 1.2. *Let $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ represent mutually distinct name variables. Then, the formula*

$$\mathcal{X}i\mathcal{Y} \wedge \mathcal{X}i\mathcal{Z} \wedge \mathcal{Y}a\mathcal{Y} \wedge \mathcal{Z}a\mathcal{Z} \rightarrow \mathcal{X}i\mathcal{X} \vee \mathcal{Y}i\mathcal{Z} \tag{1.1}$$

is unprovable in **S**.

Proof: Let $\mathcal{M} = \langle f, \mathcal{D}^a, \mathcal{D}^i \rangle$ be the model such that $f\mathcal{X} = \{1, 2\}$, $f\mathcal{Y} = \{1\}$, $f\mathcal{Z} = \{2\}$, and $\mathcal{D}^a = \mathcal{D}^i = \{\{1\}, \{2\}\}$. Then, (1.1) is false in $\mathcal{M}$, and so is unprovable by soundness. □

2. Completeness

As is ordinary in completeness proof, we begin with maximal consistency. A set Σ of formulas is called *consistent*, when for every non-empty finite subset $\{\varphi_1, \varphi_2, \ldots, \varphi_h\}$ of Σ, the formula $\neg(\varphi_1 \wedge \varphi_2 \wedge \cdots \wedge \varphi_h)$ is unprovable. It is *maximally consistent*, when it is maximal among the consistent sets.

The following lemma is commonplace.

Lemma 2.1. *For every unprovable formula, there is a maximally consistent set not containing it.*

Proof: Given an unprovable formula α, extend the consistent set $\{\neg\alpha\}$ into a maximally consistent one. □

Let Σ be a maximally consistent set. If the formula $\varphi_1 \wedge \varphi_2 \wedge \cdots \wedge \varphi_h \rightarrow \psi_1 \vee \psi_2 \vee \cdots \vee \psi_k$ is provable and $\varphi_1, \varphi_2, \ldots, \varphi_h \in \Sigma$, then at least one of $\psi_1, \psi_2, \ldots, \psi_k$ is in Σ. So, owing to the axioms, $\mathcal{X}a\mathcal{Y} \in \Sigma$ implies $\mathcal{X}i\mathcal{Y} \in \Sigma$; $\mathcal{X}i\mathcal{Y} \in \Sigma$ implies $\mathcal{Y}i\mathcal{X} \in \Sigma$; $\mathcal{X}a\mathcal{Y}, \mathcal{Y}a\mathcal{Z} \in \Sigma$ imply $\mathcal{X}a\mathcal{Z} \in \Sigma$; and $\mathcal{X}i\mathcal{Y}, \mathcal{Y}a\mathcal{Z} \in \Sigma$ imply $\mathcal{X}i\mathcal{Z} \in \Sigma$. Moreover, since $\mathcal{X}a\mathcal{Y} \rightarrow \mathcal{Y}i\mathcal{Y}$ and $\mathcal{X}a\mathcal{Y} \wedge \mathcal{X}a\mathcal{Z} \rightarrow \mathcal{Y}i\mathcal{Z}$ are provable, $\mathcal{X}a\mathcal{Y} \in \Sigma$ implies $\mathcal{Y}i\mathcal{Y} \in \Sigma$; $\mathcal{X}a\mathcal{Y}, \mathcal{X}a\mathcal{Z} \in \Sigma$ imply $\mathcal{Y}i\mathcal{Z} \in \Sigma$. Furthermore, for propositional connectives, $\neg\alpha \in \Sigma$ iff $\alpha \notin \Sigma$; $\alpha \wedge \beta \in \Sigma$, iff $\alpha \in \Sigma$ and $\beta \in \Sigma$; $\alpha \rightarrow \beta \in \Sigma$, iff $\alpha \notin \Sigma$ or $\beta \in \Sigma$; and so on.

Let Σ be a maximally consistent set. We will define the model $\mathcal{M}_\Sigma$. But, to simplify the definition, three tentative notations are introduced beforehand:

$$\mathcal{X}a\mathcal{Y} \in^* \Sigma \iff \mathcal{X}a\mathcal{Y} \in \Sigma \textit{ or } \mathcal{X} = \mathcal{Y},$$
$$\mathcal{X}i\mathcal{Y} \in^* \Sigma \iff \mathcal{X}i\mathcal{Y} \in \Sigma \textit{ or } \mathcal{X} = \mathcal{Y},$$
$$\alpha, \beta \in^* \Sigma \iff \alpha \in^* \Sigma \textit{ and } \beta \in^* \Sigma, \textit{ for atomic } \alpha \textit{ and } \beta,$$

where "$\mathcal{X} = \mathcal{Y}$" means that $\mathcal{X}$ and $\mathcal{Y}$ represent the same name variable.

Now, the model $\mathcal{M}_\Sigma = \langle f_\Sigma, \mathcal{D}^a_\Sigma, \mathcal{D}^i_\Sigma \rangle$ is defined as follows:

$$f_\Sigma \mathcal{X} = \{\langle \mathcal{U}, \mathcal{V} \rangle \mid \mathcal{U}i\mathcal{V} \in^* \Sigma \textit{ and } (\mathcal{U}a\mathcal{X} \in^* \Sigma \textit{ or } \mathcal{V}a\mathcal{X} \in^* \Sigma)\},$$

for every name variable $\mathcal{X}$, while $\mathcal{D}^a_\Sigma = \{f_\Sigma \mathcal{U} \mid \mathcal{U}a\mathcal{U} \in \Sigma\}$ and $\mathcal{D}^i_\Sigma = \{f_\Sigma \mathcal{U} \mid \mathcal{U}i\mathcal{U} \in \Sigma\}$.

The triplet $\langle f_\Sigma, \mathcal{D}^a_\Sigma, \mathcal{D}^i_\Sigma \rangle$ certainly forms a model: For, $\mathcal{X}i\mathcal{X}, \mathcal{X}a\mathcal{X} \in^* \Sigma$ imply $\langle \mathcal{X}, \mathcal{X} \rangle \in f_\Sigma \mathcal{X}$ and so $f_\Sigma \mathcal{X}$ is non-empty; while $\mathcal{D}^a_\Sigma \subseteq \mathcal{D}^i_\Sigma$, since $\mathcal{U}a\mathcal{U} \in \Sigma$ implies $\mathcal{U}i\mathcal{U} \in \Sigma$. Besides, corresponding to the axioms, $\mathcal{X}a\mathcal{Y} \in^* \Sigma$ implies $\mathcal{X}i\mathcal{Y} \in^* \Sigma$; $\mathcal{X}i\mathcal{Y} \in^* \Sigma$ implies $\mathcal{Y}i\mathcal{X} \in^* \Sigma$; $\mathcal{X}a\mathcal{Y}, \mathcal{Y}a\mathcal{Z} \in^* \Sigma$ imply $\mathcal{X}a\mathcal{Z} \in^* \Sigma$; $\mathcal{X}i\mathcal{Y}, \mathcal{Y}a\mathcal{Z} \in^* \Sigma$ imply $\mathcal{X}i\mathcal{Z} \in^* \Sigma$. Moreover, $\mathcal{X}a\mathcal{Y}, \mathcal{X}a\mathcal{Z} \in^* \Sigma$ imply $\mathcal{Y}i\mathcal{Z} \in^* \Sigma$.

Lemma 2.2. *Supposing that Σ is maximally consistent, the following properties hold:*

(1) *If $f_\Sigma \mathcal{X} \subseteq f_\Sigma \mathcal{Y}$, then $\mathcal{X}a\mathcal{Y} \in^* \Sigma$.*
(2) *If $f_\Sigma \mathcal{X} \in \mathcal{D}^a_\Sigma$, then $\mathcal{X}a\mathcal{X} \in \Sigma$.*
(3) *If $f_\Sigma \mathcal{X} \in \mathcal{D}^i_\Sigma$, then $\mathcal{X}i\mathcal{X} \in \Sigma$.*

Proof: (1) Suppose $f_\Sigma \mathcal{X} \subseteq f_\Sigma \mathcal{Y}$. Then $\langle \mathcal{X}, \mathcal{X} \rangle \in f_\Sigma \mathcal{X} \subseteq f_\Sigma \mathcal{Y}$, and so $\mathcal{X}a\mathcal{Y} \in^* \Sigma$.

(2) Suppose $f_\Sigma \mathcal{X} \in \mathcal{D}^a_\Sigma$. Then $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{U}$ for some $\mathcal{U}$ such that $\mathcal{U}a\mathcal{U} \in \Sigma$. So $\mathcal{X}a\mathcal{U}, \mathcal{U}a\mathcal{X} \in^* \Sigma$ by (1), and so either $\mathcal{X}a\mathcal{U}, \mathcal{U}a\mathcal{X} \in \Sigma$ or $\mathcal{X} = \mathcal{U}$. Hence $\mathcal{X}a\mathcal{X} \in \Sigma$ in both cases.

(3) Suppose $f_\Sigma \mathcal{X} \in \mathcal{D}^i_\Sigma$. Then $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{U}$ for some $\mathcal{U}$ such that $\mathcal{U}i\mathcal{U} \in \Sigma$. So $\mathcal{U}a\mathcal{X} \in^* \Sigma$ by (1), and so $\mathcal{X}i\mathcal{X} \in \Sigma$. □

Lemma 2.3. *Supposing that Σ is maximally consistent, $\mathcal{M}_\Sigma \models \mathcal{X}a\mathcal{Y}$ if and only if $\mathcal{X}a\mathcal{Y} \in \Sigma$.*

Proof: By definition, $\mathcal{M}_\Sigma \models \mathcal{X}a\mathcal{Y}$ iff the following three conditions (2.1)–(2.3) hold:

$$f_\Sigma \mathcal{X} \subseteq f_\Sigma \mathcal{Y}, \tag{2.1}$$

$$f_\Sigma \mathcal{Y} \in \mathcal{D}^i_\Sigma, \tag{2.2}$$

$$f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y} \textit{ implies } f_\Sigma \mathcal{Y} \in \mathcal{D}^a_\Sigma. \tag{2.3}$$

The 'only-if' part: Suppose $\mathcal{M}_\Sigma \models \mathcal{X}a\mathcal{Y}$. From (2.1) and Lemma 2.2 (1), it follows $\mathcal{X}a\mathcal{Y} \in^* \Sigma$, that is, either $\mathcal{X}a\mathcal{Y} \in \Sigma$ or $\mathcal{X} = \mathcal{Y}$. The proof is over in the former case. In the latter case, $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y}$, so $f_\Sigma \mathcal{Y} \in \mathcal{D}^a_\Sigma$ by (2.3), and so $\mathcal{Y}a\mathcal{Y} \in \Sigma$ by Lemma 2.2 (2), and hence $\mathcal{X}a\mathcal{Y} \in \Sigma$.

The 'if' part: Suppose $\mathcal{X}a\mathcal{Y} \in \Sigma$. We will show (2.1)–(2.3).

(2.1) Let $\langle \mathcal{U}, \mathcal{V} \rangle$ be any element of $f_\Sigma \mathcal{X}$. Then $\mathcal{U}i\mathcal{V} \in^* \Sigma$. Moreover, either $\mathcal{U}a\mathcal{X} \in^* \Sigma$ or $\mathcal{V}a\mathcal{X} \in^* \Sigma$; so either $\mathcal{U}a\mathcal{Y} \in \Sigma$ or $\mathcal{V}a\mathcal{Y} \in \Sigma$ by the assumption $\mathcal{X}a\mathcal{Y} \in \Sigma$. So, $\langle \mathcal{U}, \mathcal{V} \rangle \in f_\Sigma \mathcal{Y}$, and hence $f_\Sigma \mathcal{X} \subseteq f_\Sigma \mathcal{Y}$.

(2.2) $\mathcal{Y}i\mathcal{Y} \in \Sigma$ immediately follows from $\mathcal{X}a\mathcal{Y} \in \Sigma$, and so $f_\Sigma \mathcal{Y} \in \mathcal{D}^i_\Sigma$.

(2.3) Suppose $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y}$. Then $\mathcal{Y}a\mathcal{X} \in^* \Sigma$ by Lemma 2.2 (1). This together with $\mathcal{X}a\mathcal{Y} \in \Sigma$ implies $\mathcal{Y}a\mathcal{Y} \in \Sigma$, and so $f_\Sigma \mathcal{Y} \in \mathcal{D}^a_\Sigma$. □

LEMMA 2.4. *Supposing that Σ is maximally consistent, $\mathcal{M}_\Sigma \models \mathcal{X}i\mathcal{Y}$ if and only if $\mathcal{X}i\mathcal{Y} \in \Sigma$.*

PROOF: By definition, $\mathcal{M}_\Sigma \models \mathcal{X}i\mathcal{Y}$ iff the following two conditions (2.4) and (2.5) hold:

$$f_\Sigma \mathcal{X} \cap f_\Sigma \mathcal{Y} \neq \emptyset, \tag{2.4}$$

$$f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y} \textit{ implies } f_\Sigma \mathcal{Y} \in \mathcal{D}^i_\Sigma. \tag{2.5}$$

The 'only-if' part: Suppose $\mathcal{M}_\Sigma \models \mathcal{X}i\mathcal{Y}$. By (2.4), $\langle \mathcal{U}, \mathcal{V} \rangle \in f_\Sigma \mathcal{X} \cap f_\Sigma \mathcal{Y}$ for some $\mathcal{U}$ and $\mathcal{V}$. Then, $\mathcal{U}i\mathcal{V} \in^* \Sigma$, either $\mathcal{U}a\mathcal{X} \in^* \Sigma$ or $\mathcal{V}a\mathcal{X} \in^* \Sigma$, and either $\mathcal{U}a\mathcal{Y} \in^* \Sigma$ or $\mathcal{V}a\mathcal{Y} \in^* \Sigma$. In any case, it follows $\mathcal{X}i\mathcal{Y} \in^* \Sigma$, that is, either $\mathcal{X}i\mathcal{Y} \in \Sigma$ or $\mathcal{X} = \mathcal{Y}$. The proof is over in the former case. In the latter case, $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y}$, so $f_\Sigma \mathcal{Y} \in \mathcal{D}^i_\Sigma$ by (2.5), and so $\mathcal{Y}i\mathcal{Y} \in \Sigma$ by Lemma 2.2 (3), and hence $\mathcal{X}i\mathcal{Y} \in \Sigma$.

The 'if' part: Suppose $\mathcal{X}i\mathcal{Y} \in \Sigma$. We will show (2.4) and (2.5).

(2.4) The assumption $\mathcal{X}i\mathcal{Y} \in \Sigma$ together with $\mathcal{X}a\mathcal{X}, \mathcal{Y}a\mathcal{Y} \in^* \Sigma$ implies $\langle \mathcal{X}, \mathcal{Y} \rangle \in f_\Sigma \mathcal{X} \cap f_\Sigma \mathcal{Y}$. Hence $f_\Sigma \mathcal{X} \cap f_\Sigma \mathcal{Y} \neq \emptyset$.

(2.5) Suppose $f_\Sigma \mathcal{X} = f_\Sigma \mathcal{Y}$. Then $\mathcal{X}a\mathcal{Y} \in^* \Sigma$ by Lemma 2.2 (1). This together with the assumption $\mathcal{X}i\mathcal{Y} \in \Sigma$ implies $\mathcal{Y}i\mathcal{Y} \in \Sigma$, and hence $f_\Sigma \mathcal{Y} \in \mathcal{D}^i_\Sigma$. □

We immediately obtain the following lemma by induction on the construction of α, with Lemmas 2.3 and 2.4 as its basis step.

Lemma 2.5. *Supposing that* Σ *is maximally consistent,* $\mathcal{M}_\Sigma \models \alpha$ *if and only if* $\alpha \in \Sigma$*, for every formula* α. □

Theorem 2.6 (Completeness). *If a formula is true in every model, then it is provable in* **S**.

Proof: To show the contraposition, suppose that α is unprovable. Then by Lemma 2.1, $\alpha \notin \Sigma$ for some maximal consistent set Σ. So, α is false in the model $\mathcal{M}_\Sigma$ by Lemma 2.5. □

3. Kulicki's semantics

Kulicki describes two versions of model for Słupecki's system. The first version of his model is the quadruplet $\langle \mathcal{B}, f, g, \mathcal{I} \rangle$, where $\mathcal{B}$ is a non-empty family of non-empty sets, f and g are functions from the set of name variables to $\mathcal{B}$ and to the set $\{0, 1\}$, respectively, and $\mathcal{I}$ is a function whose arguments are the functions f and g and its value is the set of atomic formulas such that:

$$\begin{aligned} \mathcal{X}a\mathcal{Y} \in \mathcal{I}fg \quad &\iff \quad f\mathcal{X} \subset f\mathcal{Y} \\ &\qquad\ \textit{or } (f\mathcal{X} = f\mathcal{Y} \textit{ and } g\mathcal{X} = g\mathcal{Y} = 1), \\ \mathcal{X}i\mathcal{Y} \in \mathcal{I}fg \quad &\iff \quad (f\mathcal{X} \neq f\mathcal{Y} \textit{ and } f\mathcal{X} \cap f\mathcal{Y} \neq \emptyset) \\ &\qquad\ \textit{or } (f\mathcal{X} = f\mathcal{Y} \textit{ and } |f\mathcal{X}| \geq 2) \\ &\qquad\ \textit{or } (f\mathcal{X} = f\mathcal{Y} \textit{ and } g\mathcal{X} = g\mathcal{Y} = 1). \end{aligned}$$

The notion of truth in a model $\mathcal{M} = \langle \mathcal{B}, f, g, \mathcal{I} \rangle$ of a formula α is defined as follows: $\mathcal{M} \models \alpha$ iff $\alpha \in \mathcal{I}fg$, for atomic α; and extend the definition to compound formulas as usual.

Now, recall the formula (1.1) mentioned in Example 1.2.

Example 3.1. *The formula* (1.1) *is true in each of the first version of Kulicki's model.*

Proof: Let $\mathcal{M} = \langle \mathcal{B}, f, g, \mathcal{I} \rangle$ be an arbitrary model of the first version. We suppose $\mathcal{M} \models \mathcal{X}i\mathcal{Y} \wedge \mathcal{X}i\mathcal{Z} \wedge \mathcal{Y}a\mathcal{Y} \wedge \mathcal{Z}a\mathcal{Z}$, and will derive $\mathcal{M} \models \mathcal{X}i\mathcal{X}$ or $\mathcal{M} \models \mathcal{Y}i\mathcal{Z}$. From $\mathcal{M} \models \mathcal{X}i\mathcal{Y} \wedge \mathcal{X}i\mathcal{Z}$, it follows $a \in f\mathcal{X} \cap f\mathcal{Y}$ and $b \in f\mathcal{X} \cap f\mathcal{Z}$ for some a and b. If $a \neq b$, then $|f\mathcal{X}| \geq 2$, and so $\mathcal{M} \models \mathcal{X}i\mathcal{X}$.

So, we suppose $a = b$. Then it follows $f\mathcal{Y} \cap f\mathcal{Z} \neq \emptyset$. On the other hand, from $\mathcal{M} \models \mathcal{Y}a\mathcal{Y} \wedge \mathcal{Z}a\mathcal{Z}$, it follows $g\mathcal{Y} = g\mathcal{Z} = 1$. Hence $\mathcal{M} \models \mathcal{Y}i\mathcal{Z}$ whether $f\mathcal{Y} = f\mathcal{Z}$ or not. □

According to Kulicki [1, Theorem 3], which asserts the soundness and completeness of **S** with respect to his semantics, Example 3.1 implies that (1.1) is provable in **S**. But this contradicts Example 1.2, and so the completeness part of the theorem fails to hold. The source of this error is his deriving $|f_1\mathcal{X}| = |f_2\mathcal{X}| = 1$ from $\mathcal{M}_i \not\models \alpha \to \mathcal{X}i\mathcal{X}$ in the proof of the theorem. Instead, what we can derive is $|f_i\mathcal{X}| = 1$ alone.

Next, the definition of the second version of Kulicki's model is obtained from that of the first version by modifying two points: $\mathcal{B}$ is a non-empty family of arbitrary (possibly empty) sets; and

$$\mathcal{X}a\mathcal{Y} \in \mathcal{I}fg \iff \emptyset \neq f\mathcal{X} \subset f\mathcal{Y} \text{ or } (f\mathcal{X} = f\mathcal{Y} \text{ and } g\mathcal{X} = g\mathcal{Y} = 1).$$

Example 3.2. *The formula* (1.1) *is true in each of the second version, too.*

Proof: Let $\mathcal{M} = \langle \mathcal{B}, f, g, \mathcal{I} \rangle$ be an arbitrary model of the second version, and suppose $\mathcal{M} \models \mathcal{X}i\mathcal{Y} \wedge \mathcal{X}i\mathcal{Z} \wedge \mathcal{Y}a\mathcal{Y} \wedge \mathcal{Z}a\mathcal{Z}$. If $f\mathcal{X}$ is empty, it follows $f\mathcal{Y} = f\mathcal{Z}$ $(= \emptyset)$ and $g\mathcal{Y} = g\mathcal{Z} = 1$ from $\mathcal{M} \models \mathcal{X}i\mathcal{Y} \wedge \mathcal{X}i\mathcal{Z}$, and so $\mathcal{M} \models \mathcal{Y}i\mathcal{Z}$. In the case where $f\mathcal{X}$ is non-empty, $\mathcal{M} \models \mathcal{X}i\mathcal{X}$ or $\mathcal{M} \models \mathcal{Y}i\mathcal{Z}$ by the argument given for Example 3.1. □

References

[1] P. Kulicki, *On a minimal system of Aristotle's Syllogistic*, **Bulletin of the Section of Logic** 40 : 3/4 (2011), pp. 129–145.

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