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PARACONSISTENT HEAP. A HIERARCHY OF mbC^n-SYSTEMS

Abstract

There is no consensus on what paraconsistency is. There are several definitions of paraconsistent logic. One of them is that paraconsistent logic is a logic which is not closed under the rule P, $\neg P$ / Q (known as the rule of explosion). The pair of the formulas P and $\neg P$ entails any Q. But this definition is too broad because it includes some logics which have nothing in common with paraconsistency (provided that someone else would not read it differently from the intentions of their authors).[1]

In 2007, Carnielli, Coniglio and Marcos presented a study on the Logics of Formal Inconsistency. The study begins with a logical system called bC that is the basic logic of inconsistency.

Here we focus on a system weaker than bC (mbC for short) which appears as the result of deleting the law of double negation from the set of axioms of bC and present an axiomatization of mbC formulated directly in the language of classical propositional logic. This makes the connective of consistency redundant and enables to build a hierarchy of mbC^n-systems in which (different variants of) the rule of explosion play(s) the key role.

Keywords: paraconsistent logic, logic of formal inconsistency, mbC.

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[1]Cf. Elias H. Alves, The first axiomatization of a paraconsistent logic, Bulletin of the Section of Logic, 21(1), 1992, pp. 19–20.

1 Introduction

The language of mbC as defined in [3] is based on three classical connectives $\{\wedge, \vee, \rightarrow\}$, paraconsistent negation $\{\neg\}$ and a unary connective of consistency $\{\circ\}$. The system is axiomatizable by the negation free fragment of classical propositional logic, the principle of excluded middle and additional axiom schema $\circ P \rightarrow (P \rightarrow (\neg P \rightarrow Q))$. The sole rule of inference is detachment (ibid., p. 31).

We will present here an axiomatization of mbC formulated directly in the language of classical propositional logic. This has some consequences. For example, mbC is not the the basic logic of inconsistency anymore. Neither can it be extended by adding the principle of double negation.

The hierarchy starts with mbC^1 that is the strongest system in the hierarchy and essentially coincides with mbC by Carnielli, Coniglio and Marcos.

2 System mbC^1

Let $PROP$ denote a non-empty denumerable set of all propositional variables $\{p_1, p_2, p_3, ...\}$. The set of formulas For is inductively defined (in Backus-Naur Form) as follows:

$$\varphi ::= p_i \mid \neg P \mid P \vee Q \mid P \wedge Q \mid P \rightarrow Q,$$

where $p_i \in PROP$ and $i \in N$; P, Q are formulas and the symbols $\neg$, $\vee$, $\wedge$, $\rightarrow$ denote negation, disjunction, conjunction and implication, respectively.[2]

Definition 2.1 A truth assignment v is a function

$$v : PROP \longrightarrow \{t, f\}$$

where $\{t, f\}$ denotes the set of the logical values.

For any function $v : PROP \longrightarrow \{t, f\}$, we define $v^* : For \longrightarrow \{t, f\}$ by the induction on the degree of formulas in the following way:

$(PROP)$ $v^*(p_i) = v(p_i)$ if $p_i \in PROP$ and $i \in N$
$(\neg f)$ if $v^*(\neg P) = f$ then $v^*(P) = t$

[2] $P \leftrightarrow Q =_{df} (P \rightarrow Q) \wedge (Q \rightarrow P)$.

($\neg\neg t$) if $v^*(\neg\neg P) = t$ then $(v^*(P) = f$ or $v^*(\neg P) = f)$
($\vee$) $v^*(P \vee Q) = t$ iff $v^*(P) = t$ or $v^*(Q) = t$
($\wedge$) $v^*(P \wedge Q) = t$ iff $v^*(P) = t$ and $v^*(Q) = t$
($\rightarrow$) $v^*(P \rightarrow Q) = t$ iff $v^*(P) = f$ or $v^*(Q) = t$.

DEFINITION 2.2 (i) A mbC^1–model for the formula P is any $v : PROP \longrightarrow \{t, f\}$ such that $v^*(P) = t$.

(ii) A mbC^1–counter-model for P is a function $v : PROP \longrightarrow \{t, f\}$ such that $v^*(P) = f$.

DEFINITION 2.3 For any $P \in For$ and $\Gamma \subset For$,

P is a semantic consequence of Γ ($\Gamma \models P$, in symbols) *iff* there is no mbC^1–model in which for all formulas $Q \in \Gamma$, $v^*(Q) = t$ and $v^*(P) = f$.

DEFINITION 2.4 For any formula P,

P is a mbC^1–tautology ($\models P$, in symbols) *iff* all truth assignments v are mbC^1–models for P.

It is noteworthy at this point that the so-called *Duns Scotus' law*

(DS) $p_1 \rightarrow (\neg p_1 \rightarrow p_2)$

is not a mbC^1–tautology, whereas the formula $p_1 \rightarrow (\neg p_1 \rightarrow (\neg\neg p_1 \rightarrow p_2))$ is. The triple of the formulas P, $\neg P$ and $\neg\neg P$ entails any Q and results in the overfilling of mbC^1.

Below are the nine axiom schemata of mbC^1 which coincide with the negation free fragment of classical propositional logic (CL^+ for short), viz.

($A1$) $P \rightarrow (Q \rightarrow P)$
($A2$) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
($A3$) $((P \rightarrow Q) \rightarrow P) \rightarrow P$
($A4$) $(P \wedge Q) \rightarrow P$
($A5$) $(P \wedge Q) \rightarrow Q$
($A6$) $P \rightarrow (Q \rightarrow (P \wedge Q))$
($A7$) $P \rightarrow (P \vee Q)$
($A8$) $Q \rightarrow (P \vee Q)$
($A9$) $(P \rightarrow R) \rightarrow ((Q \rightarrow R) \rightarrow (P \vee Q \rightarrow R))$,

plus two additional

(ExM) $P \vee \neg P$

(DS^2) $P \to (\neg P \to (\neg\neg P \to Q))$

which constitute the negation fragment of mbC^1. The sole rule of inference is *Detachment Rule*, i.e. (MP) $P \to Q,\ P\ /\ Q$.

Definition 2.5 A formal proof (deduction) within mbC^1 is a finite sequence of formulas, $Q_1, Q_2, ...Q_n$, each of which is an axiom of mbC^1 or follows from the preceding formulas in the sequence by (MP). This sequence is a proof (deduction) for P if $Q_n = P$.

Definition 2.6 A formula P is a syntactic consequence within mbC^1 of a set formulas of Γ (or simply: P is provable from Γ, $\Gamma \vdash P$, in symbols) *iff* there is a formal proof in mbC^1 of P from the set Γ.

Theorem 2.1 (Soundness and Completeness) Let $\Gamma \subset For$ and $P \in For$, $\Gamma \vdash P$ *iff* $\Gamma \models P$.

An outline of the proof (for k = 2 in (DS^k)) is given in the next section.

Notice that mbC^1 is an extension of $CLuN$. The paraconsistent logic $CLuN$ (also known under the name PI) was introduced for the first time by Batens.[3] Clearly $mbC^1 = CLuN \cup \{(DS^2)\}$.

A question arises as to whether a logic that contains (DS^2) as a thesis is paraconsistent at all (cp. [7] pp. 40-41). If we answer in the negative, then, we might be tempted to ask another important question: Is there any X such that $mbC^1 \supset X \supset CLuN$ and X is a paraconsistent logic?

The general problem of paraconsistency is to determine the critical point when exactly - in Feyerabend's words - 'anything goes'; or to specify how many 'destructive' formulas (i.e. P, $\neg P$, $\neg\neg P$, $\neg\neg\neg P$, $\neg\neg\neg\neg P$, etc.) is needed to accept any Q.

3 A Hierarchy of the mbC^n-systems

In this section, we present a hierarchy of mbC^n-systems, where $n \in N$, and briefly discuss the problem of paraconsistency.

There are several hierarchies of paraconsistent logic among which da Costa's hierarchy of C-systems is probably the most famous one. The system C_ω is the weakest system in the hierarchy. C_ω is axiomatized by the

[3]See [1], pp. 204-205 and [2] p. 3.

positive fragment of intuitionistic logic, (ExM), the law of double negation $\neg\neg P \to P$ and *Detachment Rule*.[4]

As we will see, the hierarchy that we present here differs in many respects from da Costa's hierarchy of C-systems.

Let $\neg^k P = \overbrace{\neg ... \neg}^{k} P$ and $k \in N - \{1\}$.

Definition 3.1 A model (mbC^n-model) is a pair $< W, v >$, where $W = \{x\}$ and a function, $v^* : For \times \{x\} \longrightarrow \{t, f\}$, is inductively defined:

$(\neg)$ if $v^*(\neg P, x) = f$ then $v^*(P, x) = t$
$(\neg^k)$ if $v^*(\neg^k P, x) = t$ then $v^*(P, x) = f$ or $v^*(\neg P, x) = f$ or ...
... or $v^*(\neg^{k-1} P, x) = f$
$(\vee)$ $v^*(P \vee Q, x) = t$ *iff* $v^*(P, x) = t$ or $v^*(Q, x) = t$
$(\wedge)$ $v^*(P \wedge Q, x) = t$ *iff* $v^*(P, x) = t$ and $v^*(Q, x) = t$
$(\to)$ $v^*(P \to Q, x) = t$ *iff* $v^*(P, x) = f$ or $v^*(Q, x) = t$.

$\models_{mbC^n} P$ *iff* $v^*(P, x) = t$, for any mbC^n–model, any v^* and any $P \in For$.

Definition 3.2 Let $P \in For$ and $\Gamma \subset For$, $\Gamma \models_{mbC^n} P$ *iff* there is no mbC^n–model in which for all $Q \in \Gamma$: $v^*(Q, x) = t$ and $v^*(P, x) = f$.

We now provide a syntactic characterization of mbC^n.

$(A1)$ $P \to (Q \to P)$
$(A2)$ $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
$(A3)$ $((P \to Q) \to P) \to P$
$(A4)$ $(P \wedge Q) \to P$
$(A5)$ $(P \wedge Q) \to Q$
$(A6)$ $P \to (Q \to (P \wedge Q))$
$(A7)$ $P \to (P \vee Q)$
$(A8)$ $Q \to (P \vee Q)$
$(A9)$ $(P \to R) \to ((Q \to R) \to (P \vee Q \to R))$
(ExM) $P \vee \neg P$
(DS^k) $P \to (\neg P \to (\neg^2 P \to (... \to (\neg^k P \to Q))))$
(MP) $P \to Q,\ P \ /\ Q$.

Note that $k = n + 1$.

[4]See [5], for details.

Hilbert-style systems are usually awkward to handle with deriving some formula. The use of the deduction theorem leads to much shorter proofs than would be without it.

Theorem 3.1 $\Gamma \vdash_{mbC^n} P \to Q$ iff $\Gamma \cup \{P\} \vdash_{mbC^n} Q$,
where P, $Q \in For$, $\Gamma \subset For$.

Proof. Notice that any mbC^n–system is closed under $(A1)$, $(A2)$ and $P \to P$. (MP) is the sole rule of inference of any mbC^n. Then we may proceed as in the classical case.[5]

Fact 3.1 Let $\Gamma \subset For$ and P, Q, $R \in For$,
if $\Gamma \cup \{P\} \vdash_{mbC^n} R$ and $\Gamma \cup \{Q\} \vdash_{mbC^n} R$ then $\Gamma \cup \{P \vee Q\} \vdash_{mbC^n} R$.

Proof. Assume that $\Gamma \cup \{P\} \vdash_{mbC^n} R$ and $\Gamma \cup \{Q\} \vdash_{mbC^n} R$. Therefore $\Gamma \vdash_{mbC^n} P \to R$ and $\Gamma \vdash_{mbC^n} Q \to R$ by Theorem 3.1. Since $\emptyset \vdash_{mbC^n} (P \to R) \to ((Q \to R) \to (P \vee Q \to R))$ and $\emptyset \subset \Gamma$ then $\Gamma \vdash_{mbC^n} (P \to R) \to ((Q \to R) \to (P \vee Q \to R))$. It means that $\Gamma \vdash_{mbC^n} \{P \to R, Q \to R, (P \to R) \to ((Q \to R) \to (P \vee Q \to R))\}$. Apply the rule of (MP) twice to receive $\Gamma \vdash_{mbC^n} P \vee Q \to R$ and finally, $\Gamma \cup \{P \vee Q\} \vdash_{mbC^n} R$ by Theorem 3.1.

Theorem 3.2 (Soundness) Let $\Gamma \subset For$ and $P \in For$,
if $\Gamma \vdash_{mbC^n} P$ then $\Gamma \models_{mbC^n} P$.

Proof. By induction.

Theorem 3.3 (Completeness) Let $\Gamma \subset For$ and $P \in For$,
if $\Gamma \models_{mbC^n} P$ then $\Gamma \vdash_{mbC^n} P$

Proof. Assume that $\Gamma \models_{mbC^n} P$ and $\Gamma \nvdash_{mbC^n} P$. If $\Gamma \nvdash_{mbC^n} P$ then there exists $\Delta \subset For$ such that

(i) $P \notin \Delta$;
(ii) Δ is deductively closed, i.e. $\Delta \vdash_{mbC^n} Q$ *iff* $Q \in \Delta$;
(iii) Δ is relatively maximal with respect to the formula P, i.e if $Q \notin \Delta$ then $\Delta \cup \{Q\} \vdash_{mbC^n} P$, for any $Q \in For$ such that $\Delta \nvdash_{mbC^n} Q$;
(iv) $\Gamma \subset \Delta$.

[5] See [6] for details.

To show that the Lindenbaum-Asser theorem holds, define an enumeration $\{\varphi_i\}_{i\in N}$ of the formulas in the language of mbC^n and a chain of the set of formulas (theories) Δ_i, where $i \in N$, in the following way

$$\Delta_0 = \Gamma$$

$$\Delta_1 = \{\varphi_i : \Delta_0 \vdash \varphi_i\}$$

$$\vdots$$

$$\Delta_{k+1} = \begin{cases} \Delta_k \cup \{\varphi_k\}, & \text{if } \Delta_k, \varphi_k \nvdash P \\ \Delta_k, & \text{otherwise.} \end{cases}$$

Let $\Delta = \Delta_0 \cup \Delta_1 \cup ... \cup \Delta_k \cup ...$ and $i \in N \cup \{0\}$. We prove
(i) by the construction of Δ_i;
(ii) by the construction of Δ_i and transitivity of $\vdash_{mbC^n}$;
(iii) by the definition of Δ and the fact that any relatively maximal set of formulas Δ_i is a closed theory;
(iv) by the construction.

In what follows, we will need an auxiliary lemma to be proved.

Lemma 3.1 Let $\Delta \subset For$ and $Q, R \in For$,
(v) if $\neg Q \notin \Delta$ then $Q \in \Delta$
(vi) if $\neg^k Q \in \Delta$ then $Q \notin \Delta$ or $\neg Q \notin \Delta$ or ... or $\neg^{k-1} Q \notin \Delta$
(vii) $Q \vee R \in \Delta$ *iff* $Q \in \Delta$ or $R \in \Delta$
(viii) $Q \wedge R \in \Delta$ *iff* $Q \in \Delta$ and $R \in \Delta$
(ix) $Q \rightarrow R \in \Delta$ *iff* $Q \notin \Delta$ or $R \in \Delta$.

Proof. We limit ourselves to proving that the items (v) and (vi) hold.

(v) Suppose that $\neg Q \notin \Delta$ and $Q \notin \Delta$. It means that $\Delta \cup \{Q\} \vdash_{mbC^n} P$ and $\Delta \cup \{\neg Q\} \vdash_{mbC^n} P$ by (iii). Apply Fact 3.1 to receive $\Delta \cup \{Q \vee \neg Q\} \vdash_{mbC^n} P$. Notice that $\emptyset \vdash_{mbC^n} Q \vee \neg Q$ (ExM) and $\emptyset \subset \Delta$ so $\Delta \vdash_{mbC^n} Q \vee \neg Q$ and as a result, $Q \vee \neg Q \in \Delta$ by (ii). But it in turn means that $\Delta \cup \{Q \vee \neg Q\} = \Delta$ and consequently $\Delta \vdash_{mbC^n} P$. Since $\Delta \vdash_{mbC^n} P$ then $P \in \Delta$ by (ii). But $P \notin \Delta$ by (i). Contradiction.

(vi) Assume, for $k \in N - \{1\}$, that $\neg^k Q \in \Delta$, $Q \in \Delta$, $\neg Q \in \Delta$, ... and $\neg^{k-1} Q \in \Delta$.

Case (a). Let $k = 2$. Then $\neg\neg Q \in \Delta$, $Q \in \Delta$ and $\neg Q \in \Delta$. This results in the following $\Delta \vdash_{mbC^1} Q$, $\Delta \vdash_{mbC^1} \neg Q$ and $\Delta \vdash_{mbC^1} \neg\neg Q$ by (ii). Observe that $\emptyset \vdash_{mbC^1} Q \to (\neg Q \to (\neg\neg Q \to P))$ (DS^2). Since $\emptyset \subset \Delta$ then $\Delta \vdash_{mbC^1} Q \to (\neg Q \to (\neg\neg Q \to P))$ and consequently $\Delta \vdash_{mbC^1} \{Q, \neg Q, \neg\neg Q, Q \to (\neg Q \to (\neg\neg Q \to P))\}$. Now apply (MP) three times to get $\Delta \vdash_{mbC^1} P$ and $P \in \Delta$ by (ii). But $P \notin \Delta$ by (i). Contradiction.

Case (b). Let $k > 2$. Then $\neg^k Q \in \Delta$, $Q \in \Delta$, $\neg Q \in \Delta$, ... and $\neg^{k-1} Q \in \Delta$. Clearly, $\Delta \vdash_{mbC^n} Q$, $\Delta \vdash_{mbC^n} \neg Q$, ... , $\Delta \vdash_{mbC^n} \neg^{k-1} Q$ and $\Delta \vdash_{mbC^n} \neg^k Q$ by (ii), where $n \in N - \{1\}$. Notice that $\emptyset \vdash_{mbC^n} Q \to (\neg Q \to (\neg^2 Q \to (... \to (\neg^k Q \to P))$ (DS^k). Since $\emptyset \subset \Delta$ then $\Delta \vdash_{mbC^n} Q \to (\neg Q \to (\neg^2 Q \to (... \to (\neg^k Q \to P))$ (DS^k) and consequently, $\Delta \vdash_{mbC^n} \{Q, \neg Q, \neg^2 Q, ... \neg^k Q, Q \to (\neg Q \to (\neg^2 Q \to (... \to (\neg^k Q \to P))\}$. Apply (MP) (k+1)-times to get $\Delta \vdash_{mbC^n} P$. By (ii): $P \in \Delta$. But $P \notin \Delta$ by (i). Contradiction.

Define the canonical mbC^n-model for $\vdash_{mbC^n}$ as $M_k =< \Delta, v^c >$, where

$$v^c(Q, \Delta) = \begin{cases} t, & \text{if } Q \in \Delta \\ f, & \text{otherwise.} \end{cases}$$

All we need now is to show that the canonical valuation satisfies the conditions $(\neg)$, $(\neg^k)$, $(\wedge)$, $(\vee)$ and $(\to)$. This is obvious due to the fact that Lemma 3.1 holds.

Since $\Delta \nvdash_{mbC^n} P$ then $P \notin \Delta$ and consequently $v(P, \Delta) = f$. It means that $\Delta \not\models_{mbC^n} P$ and in particular $\Gamma \not\models_{mbC^n} P$. But $\Gamma \models_{mbC^n} P$ (assumption). Contradiction.

What makes difference between mbC^n-systems is how many negations occur in the axiom schema (DS^k). It leads to the following results:

(i) (DS^k) is provable in any mbC^{n-1} for $k, n \in N - \{1\}$ and $k > n$. To illustrate the point, let $k = 6$ and $n = 3$. Now to show that

$$(DS^6)\ P \to (\neg P \to (\neg^2 P \to (\neg^3 P \to (... \to (\neg^6 P \to Q))...)))$$

is provable in mbC^2, apply *Deduction Theorem*, (DS^3) and (MP);

(ii) (DS^k) is not provable in any mbC^{n+1} for $k \in N - \{1\}$, $n \in N$ and $k \leq n$. This time, as an example, let $k = 3$ and $n = 4$. The set of axiom schemata of mbC^5 consists of the negation free fragment of classical propositional logic, the law of excluded middle plus

$$(DS^6)\ P \to (\neg P \to (\neg^2 P \to (\neg^3 P \to (... \to (\neg^6 P \to Q))...))).$$

The rule of inference is (MP).

To demonstrate that

(DS^3) $P \to (\neg P \to (\neg^2 P \to (\neg^3 P \to Q)))$

is not provable in mbC^5, consider the matrix

$\mathbf{M}=< \{1,2,3,4,5,6,0\}, \{1,2,3,4,5,6\}, \neg, \wedge, \vee, \to>$,

where $\{1,2,3,4,5,6,0\}$ is the set of logical values, $\{1,2,3,4,5,6\}$ is the set of designated values and the connectives $\neg, \wedge, \vee, \to$ are defined by the tables:

$\to$	1	2	3	4	5	6	0
1	1	2	3	4	5	6	0
2	1	1	3	4	5	6	0
3	1	1	1	4	5	6	0
4	1	1	1	1	5	6	0
5	1	1	1	1	1	6	0
6	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1

	$\neg$
1	2
2	3
3	4
4	5
5	6
6	0
0	1

$\wedge$	1	2	3	4	5	6	0
1	1	2	3	4	5	6	0
2	2	2	3	4	5	6	0
3	3	3	3	4	5	6	0
4	4	4	4	4	5	6	0
5	5	5	5	5	5	6	0
6	6	6	6	6	6	6	0
0	0	0	0	0	0	0	0

$\vee$	1	2	3	4	5	6	0
1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2
3	1	2	3	3	3	3	3
4	1	2	3	4	4	4	4
5	1	2	3	4	5	5	5
6	1	2	3	4	5	6	6
0	1	2	3	4	5	6	0

All axiom schemata of mbC^5 are valid in the matrix **M** and (MP) preserves validity. To show that (DS^3) is not valid in **M**, we can take $P = 3$ (or $P = 2$ or $P = 1$) and $Q = 0$. Hence (DS^3) cannot be inferred from the set of axiom schemata of mbC^5 by means of the rule of (MP).

With these examples, it is easy to see the difference between mbC^n-systems. To put it in a more general way: let $mbC^n = \{Q : \Delta \vdash_{mbC^n} Q\}$, where $\Delta \subset For$, $Q \in For$ and $n \in N$, then

FACT 3.2 $mbC^1 \supset mbC^2 \supset mbC^3 \supset ... \supset mbC^{n-1} \supset mbC^n \supset ...$

Paraconsistent logics are those that avoid the rule of explosion. Now suppose that P is someone's belief. How many *false* beliefs do we need to accept Q? - or metaphorically speaking - How many grains of sand do we need to make a *paraconsistent* heap?

Each answer is arbitrary and finds its expression in the *levels* of explosion which are required in the hierarchy to indicate when exactly each system becomes overfilled.

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