Bulletin of the **S**ection of **L**ogic
Volume 4/2 (1975), pp. 74–76
reedition 2012 [original edition, pp. 74–77]

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A DEONTIC SENTENTIAL CALCULUS WITHOUT CERTAIN PARDOXES OF THE STANDARD SYSTEM

This is an abstract of the paper presented at the XXIst Conference for the History of Logic, Cracow, April 1975.

There exist some theses in the standard system of deontic logic SDL (cf. [1], p. 122) which have paradoxical interpretations in the natural language. Among them are the following ones:

1. $CNPpOCpq$
2. $CPKpqKPpPq$
3. $CKOpOCpqPq$
4. $COpOApq$
5. $CNPpNPKpq$
6. $CPpPApq$
7. $CAOpOqCApq$
8. $CAPpPqPApq$
9. $CPKpqPApq$
10. $CPCpqCOpPq$
11. $CKPpOCpqPq$
12. $CNPpPCpq$
13. $CKOpOCpqPq$
14. $COpPApq$
15. $CAOpOqPApq$

I have formulated some postulates in [2] which one ought to fulfil if deontic logic is to be conform to the normal meaning of deontic functors in order to become useful for inquiries concerning the consequence relation between sentences about norms. In particular, it is postulated there that: 1^0 deontic functors should preserve their intensional character, 2^0 none of the paradoxical expressions 1.-15. ought to be a thesis, 3^0 the expressions:

$T1.$ $CPApqKPpPq$
$T2.$ $EOApqKKKKKKPpPNpPqPNqCNpOqCNqOpPKpq$
$T3.$ $EOKpqKOpOq$
$T4.$ $EOCpqCpOq$
$T5.$ $KPCpqCpPq$
$T6.$ $KPpNONp$

have to be accepted as theses, 4^0 an implication, say $\underline{w}$, having $PKpq$ as antecedent may be adopted iff either (a) $\underline{w}$ has one of the forms: $CPKpqPKqp$, $CPKpqCPKNpqPq$, $CPKpqCPKqNpPq$, $CPKpqCPKpNqPp$, $CPKpqCPKNqpPp$, or (b) $\underline{w}$ is obtained from any one of the expressions mentioned under (a) by a replacement of at least one occurence of the functor P according to the equivalence $EPrNONr$, or (c) $\underline{w}$ follows from any of the expressions mentioned under (a), (b) by virtue of some laws of the classical sentential calculus.

The deontic sentential calculus DSC, described below, fulfils the above postulates. It is constructed over the classical sentential calculus.

Sentential variables occuring in the expressions of DSC represent states of affaires. They may be used in arguments of deontic functors as well as in the outside of such arguments (e.g. like in $T4$). However, superpositions of deontic functor are not admitted.

We shall identify $KKwts$ with $KwKts$ in the schemata of the metalanguage.

An axiom of DSC is an expression of the $NKu_1K\ldots Ku_{n-1}u_n$ ($n = 2, 3, 4, \ldots$) which fulfills one of the following conditions at least:

a) for some $i, j \leqslant n : u_i = Nu_j$
b) for some $i, j \leqslant n : u_i NPNw$ and $u_j = NPw$
c) for some $i, j, k \leqslant n : u_i = NPNu$, $u_j = NPNw$, $u_k = NPKuw$
d) for some $i, j, k \leqslant n : u_i = PKuw$, $u_j = PKNuw$ or $u_j = PKwNu$, $u_k = NPw$ (or $u_i = PKuw$, $u_j = PKNwu$ or $u_j = PKuNw$, $u_k = NPu$).

We adopt the after-mentioned rules:

$$EN\frac{NNw}{w};\ EA\frac{Auw}{NKNuNw};\ EC\frac{Cuw}{NKuNw};\ EE\frac{Euw}{KNKuNwNKwNu};$$
$$DO\frac{Ow}{NPNw};\ DPC\frac{PCuw}{NKuNPw};\ DPNC\frac{PNCuw}{KuPNw};\ DPK\frac{PKuw, PKwu}{PKuw, PKuw};$$

$$DPNK\frac{PNKuw}{NKNPNuNPNw};\ DPE\frac{PEuw}{KNKuNPwNKwNPu};$$
$$DPNE\frac{PNEuw}{NKNKuPNwNKwPNu};\ DPA\frac{PAuw}{KKKKPuPNuPwPNwPKuw};$$
$$DPNA\frac{PNAuw}{NKKKKKKPuPNuPwPNwNKNuPNwNKNwPNuPKuw};$$
$$EK\frac{NKu_1K\dots Ku_{n-1}KNKwtu_n}{\begin{matrix}NKu_1K\dots Ku_{n-1}KNwu_n\\ NKu_1K\dots Ku_{n-1}KNtu_n\end{matrix}}.$$

The rules, except for DPK, EK, ought to be applied in the following way: If the expression to be transformed has m parts ($m = 1, 2, 3, \dots$ not necessarily proper parts) of the form shown above the line of a rule then one should change all the m parts to expressions of the form indicated under the line of the rule. Nevertheless, the rules EA, EC, EE may be applied only to parts which are neither an argument of a deontic functor nor a fragment of such an argument. The rule DPK: when an expression contains parts of the form $PKuw$ as well as parts of the form $PKwu$, all the parts have to be transformed into $PKuw$. The rule EK concerns inproper parts (the whole of the expression) to be transformed. If requires transformations of a given expression having the form shown above the line of the rule into two expressions of the form indicated under the line of the rule. When $n = 0$, the rule EK takes the form $\frac{Kwt}{w}$ on. The order in which the rules are listed here is the order of their application.

Proofs in DSC are regressive.

The proof method is identical with the decision procedure for DSC.

References

[1] Risto Hilpinem (ed), **Deontic Logic: Introductory and Systematic Readings**, Reidel, Dordrecht 1971.

[2] Leon Gumański, *Paradoksy obowiązków i dozwoleń w standardowym systemie logiki deontycznej*, **Ruch Filozoficzny**, t. 33, z. 1 (to appear).

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