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AN EXTENSION OF LEŚNIEWSKI-CURRY'S FORMAL THEORY OF SYNTACTICAL CATEGORIES ADEQUATE FOR THE CATEGORIALLY OPEN FUNCTORS

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A categorially open functor of a language L is any functor of L whose syntactical category in L ranges at each occurrence over a set of different syntactical categories admitted in L and is determined in each case effectively in function of the categories of one or several of the expressions which it taken then as arguments.

Example: The quantifiers and the identity sign of a logical language of a predicate calculus with several types of objects are categorially open functors.

It is a well known fact that Leśniewski-Curry's theory is inadequate for the categorially open functors. (See A. Tarski, The Concept of Truth in formalized Languages, 1935,§4, 11th note and A. N. Prior, Objects of Thought, 1971, ch. 4, §7).

Leśniewski-Curry's definition of the set $Cat(L)$ of syntactical categories admitted in a language L can be enriched by the introduction in the theory of (i) variable categories whose domains of variation are sets of syntactical categories fixed in advance and of (ii) an abstractor $\frown$ binding those variables. $Cat(L)$ is then the smallest set which (1) contains all primitive and variable categories and (2) is such that if α and $\beta \in Cat(L)$ and x is a variable category then $F\alpha\beta$, $(\alpha\beta)$ and $\frown xFx\alpha \in Cat(L)$, where F and () represent here respectively the operators of functionality and of application.

A variable category x is interpreted as representing any category which belongs to its domain of variation $D(x)$.

A category of the form $\hat{x}Px\alpha$ represents the syntactical category of any unary categorially open functor of L whose arguments have a category which ranges over the set $D(x)$ and whose values, for all argument b of a syntactical category $\beta \in D(x)$, is $[\beta/x]\alpha$, the category obtained by substitution of β to all free occurrence of x in α.

If $\beta \in D(x)$ the substitution $[\beta/x]\alpha$ is defined as in the λ-calculus with the obvious necessary modifications. Otherwise, it is not defined.

The new rules of contraction for syntactical categories belonging to a set $Cat(L)$ are the the following:

$RI \quad \dfrac{(\hat{x}Fx\alpha\beta)}{(F\beta[\beta/x]\alpha\beta)}$ if $\beta \in D(x)$.

$RII \quad \dfrac{\hat{x}Fx\alpha}{yFy[y/x]\alpha}$ if $D(x) = D(y), x$ is not bound in α and y has no occurrence in α.

Added to the Leśniewski's rule: $\dfrac{(F\alpha\beta\alpha)}{\beta}$ these rules of contraction allow for a new formal theory of syntactical categories τ for which fundamental meta-theorems such as a Church-Rosser's theorem, a theorem of unicity of normal form and a theorem of consistency, are valid. Therefore naturally, for all x, $D(x)$ must be correctly defined.

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