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## DEFINABILITY CRITERION FOR FUNCTIONS IN SUGIHARA ALGEBRAS

This is an abstract of the second part of a paper that will be published in **Studia Logica**.

Let  $\mathcal{A} = (A, \sim, \vee, \wedge, \rightarrow)$  be a Sugihara algebra (see [1]). For  $a \in A$ , we shall write  $a = 0$  instead of  $\sim a = a$ .

DEFINITION 1. Two sequences  $\underline{a} = (a_1, \dots, a_k)$ ,  $\underline{b} = (b_1, \dots, b_k)$  of elements of  $A$  are said to be EXTREMALLY SIMILAR, in symbols  $\underline{a} \simeq \underline{b}$ , provided that for all  $i, j$ ,  $1 \leq i, j \leq k$ ,

- (i)  $a_i = 0$  iff  $b_i = 0$ ;  $a_i < 0$  iff  $b_i < 0$ ,
- (ii)  $|a_i| \leq |a_j|$  iff  $|b_i| \leq |b_j|$ .

Let  $f$  be a variable ranging over  $k$ -ary functions on  $A$ , that is  $f : A^k \rightarrow A$ .

DEFINITION 2. A function  $f$  is said to PRESERVE SIMILARITY provided that for every  $\underline{a}, \underline{b} \in A^k$ , if  $\underline{a} \simeq \underline{b}$  then for all  $i$ ,  $1 \leq i \leq k$ ,

- (i)  $f(\underline{a}) = \underline{a}(i)$  iff  $f(\underline{b}) = \underline{b}(i)$
- (ii)  $f(\underline{a}) = \sim \underline{a}(i)$  iff  $f(\underline{b}) = \sim \underline{b}(i)$ .

THEOREM. A function  $f$  is definable (see [1]) in  $\mathcal{A}$  if and only if

- (i) for every  $\underline{a} = A^k$ ,  $f(\underline{a}) \in \{\underline{a}(1), \dots, \underline{a}(k), \sim \underline{a}(1), \dots, \sim \underline{a}(k)\}$ ,
- (ii)  $f$  preserves similarity.

## References

- [1] M. Tokarz, *Functions definable in some fragments of Sugihara algebras*, this **Bulletin**, vol. 4, no. 1 (1975), pp. 15–18.

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