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COMPLETE SYSTEMS OF INDEXICAL LOGIC*

Symbols

A ("is actual"), B ("is fictitious"), E ("exists"),
M ("determines a model"), I ("is identical with");
N ("not"), $\rightarrow$ ("only if"), $\wedge$ ("and"), $\vee$ ("or"), $\leftrightarrow$ ("if and only if"),
$\bigwedge$ ("for any"), $\bigvee$ ("for some"), $\cap$ ("the"),
Γ ("s version of"), $\vdash$ ("yields").
t, u, v: terms, or sequences thereof.
F, G, H, J: formulas, or sequences thereof.
x, y, z: variables, or sequences thereof.

Inference rules

$F \rightarrow G$

$\dfrac{F}{G}$ (modus ponens) $\qquad \dfrac{F}{\bigwedge \times F}$ (universal generalization)

Axiom schemas

1. F (where F is a tautology).
2. $u \vdash F \rightarrow uM$.
3. $uM \rightarrow (u \vdash NF \leftrightarrow Nu \vdash F) \wedge (u \vdash (F \rightarrow G) \leftrightarrow (u \vdash F \rightarrow u \vdash G)) \wedge (u \vdash (F \wedge G) \leftrightarrow u \vdash F \wedge u \vdash G) \wedge (u \vdash (F \vee G) \leftrightarrow u \vdash F \vee u \vdash G) \wedge (u \vdash (F \leftrightarrow G) \leftrightarrow (u \vdash F \leftrightarrow u \vdash G))$.
4. $(v\Gamma(u\Gamma t)Ev(v\Gamma u)\Gamma tE \rightarrow v\Gamma(u\Gamma t)I(v\Gamma u)\Gamma t) \wedge (v \vdash u \vdash F \leftrightarrow v\Gamma u \vdash F)$.
5. $(tM \rightarrow tB) \wedge (tB \rightarrow tE)$.

*This paper is not to be reviewed.

6. $tA \leftrightarrow tE \wedge NtB$.
7. $tIu \to tE \wedge uIt \wedge (tIv \leftrightarrow uIv) \wedge (tA \to uA) \wedge (tB \leftrightarrow uB) \wedge (tM \leftrightarrow uM) \wedge (t \vdash F \leftrightarrow u \vdash F) \wedge (t\Gamma vEvu\Gamma vE \to t\Gamma vIu\Gamma v)$.
8. $u\Gamma tE \to uM$.
9. $(u \vdash tE \leftrightarrow u\Gamma tE) \wedge (u \vdash tM \leftrightarrow u\Gamma tM) \wedge (u \vdash tIv \leftrightarrow u\Gamma tIu\Gamma v)$.
10. $uM \to u \vdash ((tB \to tE) \wedge (tA \leftrightarrow tE \wedge NtB)) \wedge (u \vdash (tM \cap tA) \leftrightarrow u\Gamma tIu) \wedge u \vdash (tIv \to (tA \leftrightarrow vA) \wedge (tB \to vB))$.
11. $tE \to \bigvee xtIx$ (where x is not free in t).
12. $\bigwedge x(F \to G) \leftrightarrow (F \to \bigwedge xG)$ (where x is not free in F).
13. $\bigvee xF \to N \bigwedge xNP$.
14. $\bigwedge y(yM \to (y\Gamma tE \vee tE \to x\Gamma tIt)) \wedge tE \wedge \bigwedge xF \to^x_t F$ (where y is not free in t).
15. $\bigwedge y(uM \to y \vdash F) \wedge F \to (^U_T G \leftrightarrow G)$ (where y is not free in F and either T and U are terms such that $F = TE \vee UE \to TIU$ or T and U are formulas such that $F = T \leftrightarrow U$).
16. $t_M \to t\Gamma xIx$.
17. $tI \cap xF \leftrightarrow \bigvee y(\bigwedge x(F \leftrightarrow xIy) \wedge tIy)$ (where $y \neq x$ and y is free in neither t nor F).
18. $uM \to (u \vdash \bigwedge xF \leftrightarrow \bigwedge xu \vdash F) \wedge (u \vdash \bigvee xF \leftrightarrow \bigvee xu \vdash F) \wedge (u\Gamma \cap xFE \vee \cap xu\Gamma FE \to u\Gamma \cap xFI \cap xu \vdash F)$ (where x is not free in u).
19. $N \bigwedge xF \to yE$.
20. $G \wedge (uM \to u \vdash G)$ where the following conditions are satisfied:
    (a) G is $TE \vee UE \to TIU$ if b is term-making and $T \leftrightarrow U$ if b is formula-making.
    (b) T is $\langle b\rangle^\frown x^\frown t^\frown F$ and U is $\langle b\rangle^\frown xt(^i_y)^\frown {}^x_y i(t^\frown F)$.
    (c) x through F are sequences, y is not a value of x, x is not free in a value of $t^\frown F$, and i is an argument of x.
    (d) b is a variable binder.
21. $C(\bigwedge xH) \to J \wedge (uM \to u \vdash J)$ where the following conditions are satisfied:
    (a) J is $TE \vee UE \to TIU$ if b is term-making and $T \leftrightarrow U$ if b is formula-making.
    (b) H and U are $(t_iE \vee vE \to t_iIv) \wedge (F_j \leftrightarrow G) \wedge \bigwedge y(yM \to y \vdash ((t_iEv \vee vE \to t_iIv) \wedge (F_j \leftrightarrow G)))$ and $\langle b\rangle^\frown x^\frown t(^i_v)^\frown F(j)$ respectively if i is an argument of t and j is an argument of F.

(c) H and U are $(F_j \leftrightarrow G) \wedge \bigwedge y(yM \rightarrow y \vdash (F_j \leftrightarrow G))$ and $\langle b\rangle^\frown x^\frown F(^i_G)$ respectively if t is empty and j is an argument of F.
(d) H and U are $(t_iE \vee vE \rightarrow t_iIv) \wedge \bigwedge y(yM \rightarrow y \vdash (t_iE \vee vE \rightarrow t_iIv))$ and $\langle b\rangle^\frown x^\frown t(^i_v)$ respectively if i is an argument of t and F is empty.
(e) T is $\langle b\rangle^\frown x^\frown t^\frown F$.
(f) x through F are sequences, y is not a value of x, and y is free in neither v nor G nor is a value of $t^\frown F$.
(g) b is a variable binder for which x through F are appropriate.
(h) $c(\bigwedge xH)$ is the $\bigwedge$-closure of H with respect to x.

DEFINITION. L is the theory whose inference rules and axiom schemas are listed above.

THEOREM 1. *There is an intensional semantic theory S such that the set of all L-provable formulas = the set of all S-valid formulas.*

THEOREM 2. *A particular natural deduction system N has the same theorems as L.*