Bulletin of the **S**ection of **L**ogic
Volume 5/3 (1976), pp. 84–86

reedition 2011 [original edition, pp. 84–86]

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ENE-LOGIC

(Presented at the Seminar on Mathematical Logic, Silesian University at Katowice, May 28, 1976).

Professor Roman Suszko proposed to construct and study the ENE-logic, that is, sentential calculus with identity connective $\equiv$ and truth-functional connectives of negation $\neg$ and equivalence $\Leftrightarrow$. The construction makes use of three methods applied earlier in case of the EN-logic; see [1], [2].

Let $\vdash$ be the smallest inference relation on the set of all formulas, FM, such that for all a, b, c, d in FM and all *finite* subsets X of FM the following conditions hold:

(1.1) $\vdash a \equiv a$
(1.2) $a \equiv b, c \equiv d \vdash (a \equiv c) \equiv (b \equiv d)$
(1.3) $a \equiv b, c \equiv d \vdash (a \Leftrightarrow c) \equiv (b \Leftrightarrow d)$
(1.4) $a \equiv b \vdash \neg a \equiv \neg b$

(2.1) $a \equiv b \vdash a \Leftrightarrow b$
(2.2) $a, a \Leftrightarrow b \vdash b$
(2.3) $b, a \Leftrightarrow b \vdash a$
(2.4) $a, b \vdash a \Leftrightarrow b$

(3.1) $a, \neg a \vdash b$
(3.2) if $X; a \vdash \neg b$ then $X; b \vdash \neg a$
(3.3) $\neg a \Leftrightarrow \neg b \vdash a \Leftrightarrow b$.

Let LV be the set of all functions $t : FM \to \{1, 0\}$ such that for all a, b, c, d in FM:

(4.1) $t(a \equiv a) = 1$
(4.2) if $t(a \equiv b) = t(c \equiv d) = 1$ then $t((a \equiv c) \equiv (b \equiv d)) = 1$
(4.3) if $t(a \equiv b) = t(c \equiv d) = 1$ then $t((a \Leftrightarrow c) \equiv (b \Leftrightarrow d)) = 1$
(4.4) if $t(a \equiv b) = 1$ then $t(\neg a \equiv \neg b) = 1$
(4.5) if $t(a \equiv b) = 1$ then $t(a) = t(b)$

(5.1) $t(a \Leftrightarrow b) = 1$ iff $t(a) = t(b)$
(5.2) $t(\neg a) \neq t(a)$.

A (normal) model is a pair $\underline{M} = \langle \underline{A}, F \rangle$ where $\underline{A}$ is an algebra similar to the language $\underline{L}$, $\underline{A} = \langle A, \circ, \dot{\div}, - \rangle$ and F is a subset of A such that for all a, b in A:

(6.1) $a \circ b$ is in F iff $a = b$
(6.2) $a \dot{\div} b$ is in F iff either both a, b are in F or both a, b are not in F
(6.3) $-a$ is in F iff a is not in F.

We write: $F = F_{\underline{M}}$ and $\underline{A} = alg(\underline{M})$.

THEOREM. *For every a in FM and each subset X of FM the following three conditions are equivalent:*

(A) for all t in LV : $t(a) = 1$ whenever $t(X) = \{1\}$,
(B) for every model $\underline{M}$ and all h in $Hom(\underline{L}, alg(\underline{M}))$, $h(a)$ is in $F_{\underline{M}}$ whenever $h(X)$ is included in $F_{\underline{M}}$,
(C) there exists a finite subset Y of X such that $Y \vdash a$.

One can weaken the inference relation $\vdash$ to meet intuitionistic requirements concerning connectives $\neg$ and $\Leftrightarrow$. The *ENE*-logic is suitable for detailed studies of the difference and analogy of connectives $\equiv$ and $\Leftrightarrow$.

References

[1] Aileen Michaels and Roman Suszko, *EN-logic*, **Bulletin of the Section of Logic**, vol. 3, no. 1, p. 13.

[2] Aileen Michaels and Roman Suszko, *Sentential Calculus of Identity and Negation*, submitted to **Reports on Mathematical Logic** (Cracow, Poland).

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