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Leon Gumański

AN IMPROVEMENT OF THE DEONTIC CALCULUS *DSC*

In my paper Deontic Logic without Certain Paradoxes[1] (in short DL) I presented a deductive system of the deontical sentential calculus DSC. Although none of the well-known paradoxical theses of the standard system SDL belongs to DSC, there are still – as I have noticed recently – some theses of DSC that must be regarded as paradoxical, Among them are the following:

1. $CCpPqPCpq$
2. $CCpOqOCpq$
3. $CPNCpqp$
4. $CKCpqPCqrPCpr$
5. $CKCpqOCqrOCpr$

To show that 1. (2) is paradoxical it suffices to assume that p, Pq, $PCpq$ ($p, Oq, OCpq$) are false. The thesis 3 permits to derive from a true purely denotical premise $PNCpq$ the assertoric proposition p, which may be false. Then again if p is false and $PCqr(OCqr)$ true, the thesis 4 (5) obliges to accept the conclusion $PCpr$ ($OCpr$) even when its falsehood is known. In all the five cases the paradoxicality is essentially identical with invalidity of the formula.

The undesirable theses 1 – 5 result from the acceptance of the rules DPC, $DPNC$. However, there exist still other two rules of inference in the system, which seem to me unfortunately selected, namely: DPE and $DPNE$. By means of the latter pair of rules the following equivalences can be proved in the system:

6. $EPEpqKCpPqCqPp$

[1] Studia Logica, Vol. XXXIV, No 4.

7. $EOEpqKCpOqCpOp$

Nevertheless, the thesis 6 (7) do not render the usual meaning of the phrase 'it is permissible (obligatory) that p if and only if q' which corresponds to the expression $PEpq$ ($OEpq$). In common parlance phrases of this kind indicate a sufficient and necessary condition q of permissibility of the state of affairs p. This may be easily seen when one considers such utterances as for instance:

"You are permitted to go to the cinema if and only if you do your homework"

("It is obligatory to call a fire-brigade if and only if a conflagration has started").

Sentence of this type must not be symbolized with the help of the formula $EqPp$ ($EqOp$), because they do not consist of an assertoric component q and a purely deontic part Pq (Oq) concerning unconditional permission (obligation). It is the sentence as a whole that states the existence of a norm in the given code. As a matter of fact the norm is conditional and the functor 'if and only if' which occurs there is intensional. The difference between sentences of the form $PCpq$ ($OCpq$) and $PEpq$ ($OEpq$) lies in that the former mention only sufficient conditions. The expression:

8a. $EPEpqKPCqpNPCNqp$ (8b. $EOEpqKNPCqNpNPCNqp$)

renders adequately the current meaning of sentences about equivalential permission (obligation), but it is not a thesis of DSC.

In the light of the remarks made hitherto it should be clear that the calculus DSC ought to be altered. In what follows a deductive system of an improved version of DSC, called DSC^+, will be presented.

Axioms:[2] Expressions of the form $NKu_1Ku_2K\ldots Ku_{n-1}u_n (n > 1)$ that fulfill at least one of the following conditions for some $i, j, k \leqslant n$:

(1) $u_i = Nu_j$
(2) $u_i = NPNw, u_j = NPw$

[2]Expressions that fulfill the conditions (1) – (4) are axioms in the system of the calculus DSC. Incidentally the condition (3) has been wrongly printed in DL, it out to be given such a form as pointed out above.

(3) $u_i = NPNu, u_j = NPNw, u_k = NPKuw$ or u_iNPu, $u_j = NPNw$, $u_k = NPKNuw$ ($u_k = NPKwNu$) or $u_i = NPu$, $u_j = NPw$, $u_k = NPKNuNw$

(4) $u_i = PKuw$ and either $u_j = PKNuw$ ($u_j = PKwNu$), $u_k = NPw$ or $u_j = PKuNw$ ($u_j = PKNwu$, $u_k = NPu$

(5) $u_i = PCuw, u_j = u$, $u_k = NPw$

(6) $u_i = NPCuw$, $u_j = u$, $u_k = NPNw$

(7) $u_i = NPCuNw$, $u_j = u$, $u_k = NPw$

(8) $u_i = NPCuNw$, $u_j = u$, $u_k = PNw$

(9) $u_i = Pw$, $u_j = NPCuw$, w is not a negation

(10) $u_i = NPNw$, $u_j = PCuNw$, w is not a negation

(11) $u_i = PCuw$, $u_j = PCNuw$, $u_k = NPw$

(12) $u_i = NPCuNw$, $u_j = NPCNuNw$, $u_k = PNw$

(13) $u_i = PCuw$, $u_j = NPKuw$

(14) $u_i = NPCuNw$, $u_j = NPKuw$

(15) $u_i = NPCuw$, $u_j = NPKuNw$

(16) $u_i = NPCuw$, $u_j = NPCuNw$

As rules of inference we take all the rules of the system of DSC except for DPC, $DPNC$, DPE, $DPNE$. Instead of the four rules rejected we add six new ones marked with the sign +. The new rules are to be used in the same way as DPC in the deductive system of DSC.

$EN\ \frac{NNw}{w}$; $EA\ \frac{Auw}{NKNuNw}$; $EC\ \frac{Cuw}{NKuNw}$; $EE\ \frac{Euw}{KNKuNwNKwNu}$;

$DO\ \frac{Ow}{NPNw}$; $DPNC^+\ \frac{PNCuw}{PCuNw}$; $DPCA^+\ \frac{PCAuvw}{KPCuwPCvw}$;

$DPCA-N^+\ \frac{PCAuvNw}{NKNPCuNwNPCvNw}$;

$DPC-NK^+\ \frac{PCuNKvw}{NKNPCuNvNPCuNw}$;

$DPK\ \frac{PKuw, PKwu}{PKuw, PKuw}$; $DPNK\ \frac{PNKuw}{NKNPNuNPNw}$;

$DPE^+\ \frac{PEuw}{KPCwuNPCNwu}$; $DPNE^+\ \frac{PNEuw}{NKNPCwNuNPCNwu}$;

$DPA\ \frac{PAuw}{KKKKPuPNuPwPNwPKuw}$;

$DPNA\ \frac{PNAuw}{NKKKKKKPuPNuPwPNwNKNuPNwNKNwPNuPKuw}$;

$$EK\ \frac{NKu_1K\ldots Ku_{n-1}KNKwtu_n}{\begin{array}{c}NKu_1K\ldots Ku_{n-1}KNwu_n\\ NKu_1K\ldots Ku_{n-1}KNtu_n\end{array}}.$$

In the rules $DPCA^+$, $DPCA-N^+$ the variable w do not represent negations. The order of rules is identical with the succession of their application.

The notions of expression, reduction sequence, proof, thesis and antithesis are defined just as in DL.

All the paradoxical theses of SDL as well as the paradoxical theses 1 – 5 of DSC (see above) are anti theses of DSC^+. It is worth mentioning that the expressions:

$T27a\ CNpOCpq$
$T27b\ OCNpCpq$

which are theses of DSC but were severely criticized in the past, are antitheses of the calculus DSC^+.

At the end of DL I suggested that the antitheses of DSC:

$AT17\ COCpqPKpq$
$AT17a\ CPCpqPKpq$

seem to be in full accordance with common intuition and ought to get admitted. As a matter of fact both of them are theses of DSC^+. Moreover, it should be entered to the credit of the calculus DSC^+ that the expressions 8a, 8b as well as $T38$, $T41$ (see DL) and besides also the converse implications:

9. $COCpNqONCpq$
10. $CPCpNqPNCpq$

belong to the set of theses of DSC^+ whereas 6 and 7 do not.

Chair of Logic
Nicholas Copernicus University
Toruń