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# DEDUCTIVE SYSTEMS AND CONGRUENCE IN ORTHOLATTICES

Let $L$ be ortholattice and let denote the orthocomplement. The weak implication is the binary operation $x \to y = x' \vee y$ and the strong implication is defined (cf. G. Kalmbach, *Orthomodular Logic*, **Proc. Univ. Houston Lattice Theory Conf.**, 1973, pp. 498–503) as $x \mapsto y = (y \to x) \to ((y' \to x) \to x \wedge (x \to y))$. A (strong) deductive system is a subset $D \subseteq L$ such that $1 \in D$ and if $x$ and $x \mapsto y$ are in $D$ ($x$ and $x \leftrightarrow y$ are in $D$) then $y \in D$. A Finch filter is a filter $F$ of $L$ such that if $z \in F$, then $x \to (x \wedge y) \in F$ for all $x$ in $L$. It is shown that the notions of deductive system, strong deductive system and Finch filter are mutually equivalent in every ortholattice.

A deductive system $D$ is said to be Boolean (orthomodular) if the relation $(x \to y) \wedge (y \to x) \in D$ $((x \mapsto y) \wedge (y \mapsto x) \in D)$ is a congruence on $L$, and it is shown that the Boolean (orthomodular) homomorphic images of $L$ are in one-to-one correspondence with the Boolean (orthomodular) homomorphic images of $L$ are in one-to-one correspondence with the Boolean (orthomodular) deductive systems. Moreover, $D$ is a Boolean deductive system if and only if it is an intersection of prime filters of $L$.

Finally, by using results of P. D. Finch (**J. Austral. Math. Soc.** 6 (1966), pp. 46–54) it is shown that if $L$ is an orthomodular lattice, then there is a one-to-one correspondence between deductive systems and congruence relations on $L$. In particular, it follows that if $L$ is an orthomodular lattice satisfying the chain condition, then the congruence lattice of $L$ is anti-isomorphic to the center of $L$, and therefore, it is a Boolean algebra.

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