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DUAL COUNTERPARTS OF STRONGLY FINITE CONSEQUENCES

Theorem. *There is a strongly finite consequence C and an elementary matrix $\mathcal{M}$ such that $C = Cn_{\mathcal{M}}$ and dC is not strongly finite. (In [1] Wójcicki stated a hypothesis which is the negation of the theorem. The terminology used here follows [1]).*

Proof. *Let $\mathcal{M} = ((\{0,1\}, \rightarrow), \{0\})$ be an elementary matrix where $0 \rightarrow 0 = 0 \rightarrow 1 = 1 \rightarrow 1 = 1$, $1 \rightarrow 0 = 0$. Let C be the consequence determined by this matrix. It can easily be proved that if $\alpha \rightarrow \beta \in C(\gamma)$ then $\beta \in C(\gamma)$ and if $p_1 \in C(p_2)$ then $p_1 = p_2$. Let p_1, p_2, p_3 be different variables. We prove that $C(p_1) \cap C(p_2) \subseteq C(p_3)$. Suppose that $\alpha \in C(p_1) \cap C(p_2)$. α is a formula of the form $\alpha_1 \rightarrow (\alpha_2 \rightarrow \ldots \rightarrow (\alpha_i \rightarrow p_4) \ldots)$. Then $p_4 \in C(p_1) \cap C(p_2)$. Hence $p_4 = p_1$ and $p_4 = p_2$, and we obtain a contradiction. Then $C(p_1) \cap C(p_2) = \emptyset$. So $C(p_1) \cap C(p_2) \subseteq C(p_3)$. Let e be a substitution such that $ep_1 = ep_2 = p_1$, $ep_3 = p_3$. It is easy to see that $C(ep_1) \cap C(ep_2) \not\subseteq C(ep_3)$. By (Proposition 2, Theorem 8 in) [1] dC is not strongly finite. Q.E.D.*

References

[1] R. Wójcicki, Strongly finite sentential calculi, [in:] **Selected papers on Łukasiewicz sentential calculi**, *Wrocław, 1977.*

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