Bulletin of the **S**ection of **L**ogic
Volume 7/2 (1978), pp. 86–86
reedition 2011 [original edition, pp. 86–87]

Rolando Chuaqui

BERNAYS' CLASS THEORY

This is a (mainly) expository paper about a system of axiomatic set theory introduced by P. Bernays in *Zur Frage der Unendlichkeits schemata in der Axiomatischen Mengenlehre*, **Esays on the Foundations of Mathematics**, Magnes Press, Jerusalem, 1961. This system is a theory of classes with two types of objects: classes and sets. However, we use only one type of variables and express 'x is a set' by '$\exists u(x \in u)$'. The axioms use only $\in$ as non-logical symbol and are expressed in first-order logic with identity. These axioms are: the inpredicative axiom of class specification, extensionality, axiom of subsets (i.e., subclass of a set is a set), and a reflection principle.

In Section 1 the axioms are given it is shown that they are intuitively justified. It is also proved that the theory contains MT, the impredicative theory of classes (see Kelly, *General Topology*, Appendix, for this theory).

In Section 4, results of Tharp, *On a set theory of Bernays*, **J. of Symb. Logic** 32 (1967), pp. 319–321, are given which indicate that the Π_n-indescribable cardinals are exactly those which can be obtained in the theory.

Another end of this paper is to show, by giving an example, how the methods developed in Kreisel and Lévy, *Reflection principles*, **Zeitscht. f. math. Logik und Grundl. der Math.** 14 (1968), pp. 97–112, are used to study the complexity of axiom systems. This is done in Section 3, where a detailed proof is given of the impossibility of axiomatizing the theory by adding a set of sentences of bounded quantifier depth to MT. This is, probably, the only new result of the paper.

Instituto de Matemátics
Universidad Católica de Chile
and
Departamento de Matemática
Universidade Estadual de
Campinas, Brazil