Bulletin of the **S**ection of **L**ogic
Volume 7/4 (1978), pp. 179–184

reedition 2011 [original edition, pp. 179–184]

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ON DECIDABILITY OF A SYSTEM OF DIALECTICAL PROPOSITIONAL LOGIC

1. Introduction

In a recent paper, da Costa and Wolf presented a system of propositional logic, called DL, "designed to match a particular doctrine in dialectical theory, that of the unity of opposites" (see [2]). According to them, the system is based on a discussion in McGill and Parry [3].

In this paper, we give a description of da Costa and Wolf's DL and of the corresponding two-valued semantics. Then we propose a decision method for that calculus.

2. The Dialectical Logic DL

The primitive symbols of DL are propositional variables, parentheses and connectives: $\supset$ (implication), $\wedge$ (conjunction), $\vee$ (disjunction), $\neg$ (negation) and a new primitive unary connective, $^\circ$, which serves as an indicator of stability; i.e., if A° is true, then A is "well-behaved". Many postulates are introduced just to ensure that the propositions of the form A° obey the laws of classical logic. The notion of formula is defined in the standard way.

The postulates of DL are the ones of classical positive logic and the following:

(1) $\neg(A \wedge B) \equiv (\neg A \vee \neg B)$
(2) $\neg(A \vee B) \equiv (\neg A \wedge \neg B)$
(3) $(A^\circ \wedge B^\circ) \supset ((A \supset B)^\circ \wedge (A \wedge B)^\circ \wedge (A \vee B)^\circ \wedge (\neg A)^\circ)$
(4) $(A^\circ \wedge B^\circ \supset ((A \supset B) \supset ((A \supset \neg B) \supset \neg A))$

(5) $A^\circ \supset (\neg\neg A \supset A)$
(6) $A^{\circ\circ} \equiv A^\circ$
(7) $A^\circ \supset ((A \vee \neg A) \wedge ((A \supset B) \vee (\neg A \supset B))$
(8) $\neg A^\circ \supset ((A \vee \neg A) \supset B) \vee (A \wedge \neg A))$

Definition 1 (of strong negation). $\sim A =_{def} A \supset (p^\circ \wedge p \wedge \neg p)$ where p is a fixed atomic formulas.

In DL the symbols $\wedge, \vee, \supset$ and $\equiv$ have all properties of the classical positive logic. The strong negation has all properties of classical negation.

3. A two-valued semantics for DL

The semantics used by da Costa and Wolf is the Henkin-style semantics which has proven fruitful in treating other paraconsistent logics (see [1]).

Definition 2. $\overline{\Gamma} = \{A \in F : \Gamma \vdash A\}$, where F is the set of all formulas of DL.

Definition 3. Γ is said to be *trivial* if $\overline{\Gamma} = F$; otherwise Γ is called *non-trivial*. Γ is said to be *inconsistent* of there is at least one formula A such that A, $\neg A \in \Gamma$; otherwise Γ is called *consistent*. Γ is *non-trivially maximal* if it is non-trivial and, for every formula A, if $\Gamma \vdash A$, then $A \in \Gamma$ (or $\Gamma = \overline{\Gamma}$). Γ is said to be $\neg$-*incomplete* if there is at least one formula A such that $\Gamma \nvdash A$ and $\Gamma \nvdash \neg A$; otherwise Γ is $\neg$-*complete*.

Definition 4. A valuation of DL is a function $v : F \to \{0, 1\}$ such that:

1) $v(A \supset B) = 1 \leftrightarrow v(A) = 0$ or $v(B) = 1$
2) $v(A \wedge B) = 1 \leftrightarrow v(A) = v(B) = 1$
3) $v(A \vee B) = 1 \leftrightarrow v(A) = 1$ or $v(B) = 1$
4) $v(\neg(A \wedge B)) = 1 \leftrightarrow v(\neg A) = 1$ or $v(\neg B) = 1$
5) $v(\neg(A \vee B)) = 1 \leftrightarrow v(\neg A) = v(\neg B) = 1$
6) $v(A^\circ) = v(B^\circ) = 1 \Rightarrow v((A \supset B)^\circ) = v((A \wedge B)^\circ) = v((A \vee B)^\circ) = 1$
7) $v(A^\circ) = 1 \Rightarrow v((\neg A)^\circ) = 1$
8) $v(A^\circ) = 1 \Leftrightarrow v(A^{\circ\circ}) = 1$
9) $v(A^\circ) = 1 \Rightarrow (\neg\neg A \supset A) = 1$
10) $v(A) = v(\neg A) \Rightarrow v(A^\circ) = 0$
11) $v(A) \neq v(\neg A) \Rightarrow v(\neg A^\circ) = 0$

The definitions of *Validity*, *model* and *consequence* are the standard ones.

THEOREM 1. *Every non-trivial set of formulas is contained in a maximal non-trivial set.*

THEOREM 2. *Every maximal non-trivial set of formulas has a model.*

THEOREM 3. $\Gamma \vdash A$ *iff* $\Gamma \models A$.

Note that this semantics is a straightforward generalization of that for the classical propositional calculus and makes clear the meanings of $\sim$ and $\neg$.

4. Decidability of DL

By means of the proposed semantics, we can obtain a decision method for DL, which is an adaptation of the method described in [1]. For each formula A of DL we construct tables, called *quasi-matrices*, according to the instructions below. A is a theorem of DL if and only if in any quasi-matrix for A the last column contains only $1'$s. In order to construct a quasi-matrix for A, the procedure is as follows:

1) In a line, make a list of all the subformulas of A and some others formulas, as follows:
   i) First, write every atomic formula which occurs in A.
   ii) Then, write every negation of these atomic formulas.
   iii) Write the other subformulas of A but including (if they are not yet) other formulas, in such a way that:
      1° $\neg B$ is written before B°;
      2° $\neg C$ and $\neg D$ are written before $\neg(C \vee D)$ or $\neg(C \wedge D)$;
      3° $\neg C$, $\neg D$, C° and D° are written before $\neg(C \supset D)$;
      4° A° and B° are written before $(A \supset B)^\circ$, $(A \vee B)^\circ$ and $(A \wedge B)^\circ$;
      5° B° is written before $(\neg B)^\circ$.
   iv) A formula can be written only if its proper subformulas have already been written.
2) Under the list of the atomic formulas, place in successive lines all the possible combinations of 0 and 1 which can be attributed to these

formulas.

3) Under each negation of an atomic formula, bifurcate each line, writing in the first part 0 for the negation, and, in the second part, 1. Every time there is a bifurcation, we proceed as above, and this means that the values are the same for the two new lines in the part on the left of it.
4) Calculate, in the order of the construction of the list, for each line, the value of each formula as follows:
   i) When no negations and no formula of the form B° are involved, proceed as in a truth-table for the classical calculus.
   ii) If any of the formulas under consideration is a negation and so of the form $\neg B$, proceed as follows:
      1° If B is of the form C°, check if the value of C is equal to the value of $\neg C$. If this is the case, bifurcate the line. If the value of C is different from the value of $\neg C$, simply write 0.
      2° If B is of the form $\neg C$, check if the value of C is equal to the value of $\neg C$. If this is the case, bifurcate the line. If the value of C is different from the value of $\neg C$, check if the value of C° is 0 or 1. If it is 0 bifurcate the line. If it is 1, simply write the same value of C.
      3° If B is of the form $C \supset D$ check if the value of C is equal to the value of $\neg C$, or if the value of D is equal to the value of $\neg D$. If this is true, bifurcate the line. If this is not true, check if C° or D° has value 0. If this is the case, bifurcate the line. If C° and D° have value 1, write 0 if B has value 1, and vice-versa.
      4° If B is of the form $C \vee D$, write 0 if $\neg C$ or $\neg D$ have value 0. If this is not the case, write 1.
      5° If B is of the form $C \wedge D$, write 0 if $\neg C$ has value 0 and $\neg D$ has value 0. If this is not the case, write 1.
   iii) If any of the formulas under consideration is of the form B°, check if the value of B is equal to the value of $\neg B$. If this is true, write 0 under B°. If this is not true, proceed as follows:
      1° If B is an atomic formula, bifurcate the line.
      2° If B is $\neg C$, check if C° has value 1. If this is the case, write 1 under B°. If C° has value 0, bifurcate the line.

3° If B is $C \supset D$, $C \vee D$ or $C \wedge D$, check if the values of C° and D° are 1. If this is the case, write 1 under B°. If the value of C° or the value of D° is 0, bifurcate the line.

4° If B is of the form C°, write under B° the value of C°.

References

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