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SENTENTIAL CONSTANTS IN RELEVANCE IMPLICATION (ABSTRACT)

Sentential constants have been part of the R environment since Church [1]. They have had diverse uses in explicating relevant ideas and in simplifying them technically. Of most interest have been the Ackermann pair of constants t, f, functioning conceptually as a least truth, (or, As circumstances semantically abnormal, as a least assertion) and as a greatest (or denial), under the ordering of propositions under true (or asserted) implication. Also interesting have been the Church constants F, T, functioning similarly as least greatest propositions.

The Ackermann constants have the following uses in systems of relevant implications: (i) defining negation inferentially; (ii) giving rise to theories of enthymematic implication and elliptical negation; (iii) having special significance in the propositional structures that interpret systems of relevant implication algebraically; (iv) being linked to the "real world" 0 in the model structure of a Kripke-style semantics; (v) easing various minor formal tasks, like simplifying axioms and facilitating proofs. There is rarely a good reason to omit t and f in formulating R. There are many good reasons to include them, as a basic part of the syntactical apparatus.

The Church constants, despite their priority, have been less pervasive. But they have the following uses: (i) introducing into R, and especially into $R^{\urcorner}$, an $S5$ theory of necessity, linked semantically with truth in all possible "worlds"; (ii) simplifying accordingly the R and E theories of first-degree formulas, coding them as theories of "strict implication"; (iii) introducing the notion of "rigorous compactness", which has proved helpful in various arguments involving matrix manipulation; (iv) providing R with links to, and distinctions from, intuitionist logic, on the topic of intuitionist negation.

In this paper we concentrate primarily on examining the structures to which the constants alone give rise, in the intensional part R_i of R, in R itself, and in the Boolean extension $R^{\urcorner}$ of R. On the whole, these structures are very simple. F, T, taken in isolation, give rise to the 2 element Boolean algebra. At the intensional level of R_i, f and t give rise to the same 6 element structure as does the 1 variable fragment of $R_{\rightarrow}$. When F and T are also added, this turns into a 14 element structure in R_i, on application of principles of rigorous compactness. In R itself, the structure of the Ackermann constants begins to get complicated, as compounding of the 6 intensional elements under & and $\vee$, and recompounding under $\rightarrow$, etc., produces a wealth of new elements. It is unknown whether the resulting structure is finite or infinite, in R itself, though 32 distinct formulas have been found in R, pairwise non-equivalent and involving only Ackermann constants. If the Church constants are added also, the structure becomes more complicated still, through the increased complication can be brought under control by applying again the rigorous compactness principles.

In the conservative Boolean extension $R^{\urcorner}$ of R (née CR^* in [2]), the constant structure again becomes quite simple, when t and f are added. T and F are already present in this system, by definition. Moreover, they are already definable in the intensional Ackermann-constant part of $R^{\urcorner}$, taking t as $f \rightarrow t \rightarrow t$, and F as $T \rightarrow f$. The result is that the intensional t, f part of $R^{\urcorner}$ has the same 6 formulas as before; but the t, f, T, F fragment of $R^{\urcorner}$ is the same, a reduction from the previous 14. More surprising, the full constant structure of $R^{\urcorner}$, in both Church and Ackermann constants, is the S element Boolean algebra, adding only the new constants $(f \circ f) \& t$ and its negation $(f \rightarrow t) \vee f$ to complete the intensional hexagon.

In the last part of this paper, we look at a number of further questions. We interest ourselves in the elimination by explicit or contextual definition, of the Church and Ackermann constants in systems of relevant implication. t, f are explicitly definable nowhere, while T, F are definable only at the Boolean level of $R^{\urcorner}$. Contextual definition, on the other hand, is generally possible, both for Church and Ackermann constants. We fail, however, to find any sort of definition, explicit or contextual, for the Ackermann pair f, t in $R^{\urcorner}$. After a brief glance at the role of the sentential constants in other relevant logics, such as E, we turn to questions of conservative extension, which we find motivationally important. All constants can be added conservatively to the corresponding constant-free systems. But if a positive system is formulated with constants, and then negation is added, either it is DeMorgan $\sim$ or Boolean $\neg$ form, the usual run of relevant conservative extension results is interrupted. We find the cause of this phenomenon, which does not affect constant-free formulas, in a subtle equivocation in the sense of the constants, when now particles are added. The most striking instances of the phenomenon are that the sense of F is not conserved on the passage from R_{+}^{F} to R^{F}, and that the sense of t is not conserved on the passage from R^{t} to $R^{t\urcorner}$. By introducing *restricted* constants F_{+} and t^{R}, governed in the wider system only by those axioms that fairly belong to the narrower conceptual universe, the difficulty is overcome, leading to a contrast between restricted and unrestricted constants in the wider systems. Where the constants could be understood unequivocally in passing from a narrower to a wider system, the usual conservative extension results obtain.

Why study the constants? Besides intrinsic interest, worth noting again is that the structures investigated here will show up in every model of R, algebraic or semantic. For example, prime or normal DeMorgan monoids must be rigorously compact, whenever they have top and bottom elements – e.g., always, when they are finite. All DeMorgan monoids will have elements t and f, whence the relations of the matrix 6 (and of 14, usually) must be respected. We have noted the importance of Boolean algebras among the DeMorgan monoids – for example, the structure M_0 of [3]. Such models must also reflect the tight structure of the characteristic matrix 8 for constants in $R^{\urcorner}$. So the structures of this paper will inevitably be reflected, at least homomorphically and often isomorphically, in the most important models for R and its kin.

Finally, a personal word. I set out to write this paper with the thought that, since considerations involving them arise so often, it was time for a full characterization of the sentential constants in R and its variants. I initially expected that, as in the $R^{\urcorner}$ case, the Ackermann structure would peter out in R (after adding a few more elements to overcome the non-distributivity of 6). In fact, the Magic Number seemed to be 12, and I even had a 12ist waiting in the wings (V. Meyer, the important 21ist century philosopher, who opines that 12 is better than 6 because it is more even, citing Babylonian wisdom in further suport of her views), to dispute with Belnap, A. McRobbie, and other numerologists. Alas, it was all for nought when R passed its presumptive Magic Number by without pausing for breath, and 12 (and V. Meyer) will have to try harder to catch up relevantly with 6, 8, and 14.

The personal word, however, does induce a final quandary close to which we have persistently hovered. The sentential constants are clearest in $R^{\urcorner}$, where their properties have been normalized. They are least clear in R itself. However, as we indicated in the last section, it seems that both the t of R, taken as t^R, and the t of $R^{\urcorner}$ point to concepts of which we ish to take account. A Long Run normality, symbolized by the latter, is at least a regulative ideal. But Short Run abnormalities, caught in t^R, are facts of life. There is room, in relevant thinking, both for normal ideal and facts of life. Preferring either at the expense of the other is to lose part of the story. While I believe that the whole story is most satisfactorily told in $R^{\urcorner}$, and that R itself is best viewed as a fragment of this system, it is an important fragment, even as the wholly intensional system R_i is an important fragment of R, and for similar reasons. It is no use containing relevant ideas in wider wholes unless they are also *preserved*. Otherwise, we could have stuck to truth-functional logic all along. Our apparent quandary is that we must make a choice between t^R and t. Instead, since they catch opposite sides of the relevant enterprise – the demands of practical reason on the one hand, and a normal ideal on the other, elements that intermingle in our actual reasoning behavior – let us have them both – t^R as our present Best Hypothesis, and t as the Truth to which (we hope) we shall ultimately come.

References

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