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OMEGA-CONSISTENCY AND THE DIAMOND
(Abstract)

G is the result of adjoining the schema $\Box(\Box A \to A) \to \Box A$ to K; the axioms of G^* are the theorems of G and the instances of the schema $\Box A \to A$ and the sole rule of G^* is modus ponens. A sentence is ω-provable if it is provable in P(eano) A(rithmetic) by one application of the ω-rule; equivalently, if its negation is ω-inconsistent in PA. Let ω-(Bew(x) be the natural formalization of the notion of ω-*provability*. For any modal sentence A and function ϕ mapping sentence letters to sentences of PA, inductively define $A^{\omega\phi}$ by: $p^{\omega\phi} = \phi(p)$ (p a sentence letter); $\bot^{\phi} = \bot$; $(A \to B)^{\omega\phi} = (A^{\omega\phi} \to B^{\omega\phi})$; and $(\Box A)^{\omega\phi} = \omega$-Bew$(\ulcorner A^{\omega\phi} \urcorner)$ ($\ulcorner S \urcorner$ is the numeral for the Gödel number of the sentence S). Then, applying techniques of Solovay (**Israel Journal of Mathematics** 25, pp. 287–304), we prove that for every modal sentence A, $\vdash_G A$ iff for all $\phi, \vdash_{PA} A^{\omega\phi}$; and for every modal sentence A, $\vdash_{G^*} A$ iff for all ϕ, $A^{\omega\phi}$ is true. (To appear in **Studia Logica**).

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