Bulletin of the **S**ection of **L**ogic
Volume 9/2 (1980), pp. 57–59

reedition 2010 [original edition, pp. 57–59]

Bogusław Wolniewicz

ON THE VERIFIERS OF DISJUNCTION

In a letter David Makinson has pointed out to me a clear-cut counterexample to the formula T3: $S(\alpha \vee \beta) = Min(S(\alpha) \cup S(\beta))$, propounded in [1] for objective of disjunction. As he says: let p, q be independent E-propositions and put $\alpha = p \wedge q$, $\beta = p \wedge \sim q$. Since p is an E-proposition, $S(p) = \{u\}$ for some u in SE. Then $S(\alpha \vee \beta) = S((p \wedge q) \vee (p \wedge \sim q)) = S(p) = \{u\}$; but $u \notin V(\alpha) \cup V(\beta) \supset S(\alpha) \cup S(\beta) \supset Min(S(\alpha) \cup S(\beta))$.

The formula T3 was entailed (cf. [2]) by the axiom A5: $V(\alpha \vee \beta) = V(\alpha) \cup V(\beta)$. Consequently, the essential half of A5 has to be dropped too, though by A2 we are still left with the other one:

/D1/ $$V(\alpha) \cup V(\beta) \subset V(\alpha \vee \beta).$$

However, in view of the other axioms, the following equivalence holds with regard to the verifiers of disjunction:

/D2/ $$s \in V(\alpha \vee \beta) \equiv \bigwedge_{t \geq s} \bigvee_{u \geq t} u \in V(\alpha) \cup V(\beta).$$

To see this observe that

/N/ $$s \in V(\sim \alpha) \equiv \sim \bigvee_{t} (s \leq t \wedge t \in V(\alpha))$$

is a theorem. Indeed, implication $\leftarrow$ of N is equivalent to A9. On the other hand, if $s \in V(\sim \alpha)$ and $s \leq t$, then $t \in V(\sim \alpha)$ by A3. Hence $\sim t \in V(\alpha)$ by A6, cancelling double negation. Starting now with $V(\alpha \vee \beta) = V(\sim(\sim \alpha \wedge \sim \beta))$, D2 may be derived from N by A4, exactly as the theorem 2.4/iii is in [3], if only we read Bell's "p forces σ" as "$p \in V(\sigma)$", and reverse everywhere his "$\leq$".

Let $D_1, D_2, \ldots, D_n$ be the SE-sets corresponding to the logical dimensions of a W-language L. (Their number is assumed to be finite!) Thus $D_1 \cup D_2 \cup \ldots \cup D_n = Min(SE)$, and the product $PL = D_1 \cdot D_2 \cdot \ldots \cdot D_n = Max(SE)$ is then the *logical space* of L, i.e. it is the totality of *logical points* or possible worlds of L. (Cf. [4].) We have here the lemma: $V(\alpha) \cdot PL = V(\alpha) \cap PL$. (The inclusion $\supset$ is obvious. Conversely, if $u = s; w \wedge s \in V(\alpha) \wedge w \in PL$, then $u \in V(\alpha)$ by A3, and $u \in PL$ by w being maximal). This leads to the theorem:

/D3/ $$V(\alpha \vee \beta) \cdot PL \subset V(\alpha) \cup V(\beta),$$

for suppose $w \in V(\alpha \vee \beta)$ and $w \in PL$. Then $w \in V(\alpha) \cup V(\beta)$ by D2, which in view of the lemma yields D3.

We adopt now a new axiom:

/A10/ $$A \cdot PL \subset V(\alpha) \rightarrow A \subset V(\alpha)$$

for any $A \subset SE$. (We preserve the numbering of [1], though A5 is wanting; moreover, A7 and A8 are redundant, since A8 follows from A7, and A7 is provable with the number of dimensions being finite). From this axiom we get another formula for $V(\alpha \vee \beta)$:

/D4/ $$V(\alpha \vee \beta) = \bigcup \{A \subset SE; A \cdot PL \subset V(\alpha) \cup V(\beta)\}.$$

Indeed, suppose $s \in V(\alpha \vee \beta)$. Then by D3 we have $s \in A \wedge A \cdot PL \subset V(\alpha) \cup V(\beta)$ for some A, i.e. $s \in \bigcup\{\ldots\}$. Conversely, if $s \in \bigcup\{\ldots\}$, then by D1: $s \in A \wedge A \cdot PL \subseteq V(\alpha \vee \beta)$, for some $A \subseteq SE$. Hence by A10 we get $s \in V(\alpha \vee \beta)$.

But what about the objective $S(\alpha \vee \beta)$ as a function of $V(\alpha)$ and $V(\beta)$? This will be discussed in a wider context, together with the objectives of tautology and negation.

References

[1] B. Wolniewicz, *Objectives of Propositions*, this **Bulletin**, vol. 7, no. 3 (1978), pp. 143–147.

[2] B. Wolniewicz, *Some Formal Properties of Objectives*, this **Bulletin**, vol. 8, no. 1 (1979), pp. 16–20.

[3] J. L. Bell, **Boolean-Valued Models and Independence Proofs**, Oxford 1977.

[4] R. Suszko, *Ontology in the Tractatus of L. Wittgenstein*, **Notre Dame Journal of Formal Logic**, vol. 9, no. 1 1968), p. 15.

Institute of Philosophy
Warsaw University