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# SOME EXAMPLES CONCERNING UNIFORMITY AND COMPLEXITY OF SENTENTIAL LOGICS

This abstract is a contribution to the paper by R. Wójcicki [2]. We use the terminology introduced there, (cf. also the preceding abstract [1], this volume).

In this note we show that

$1^0$ For each $n \geq 2$ there exists a logic whose degree of uniformity and degree of complexity are both equal to $n$.

$2^0$ There is a logic with the degree of uniformity equal to $2^{\aleph_0}$ but with the degree of complexity equal to $\aleph_0$.

Let $L$ be a language built up from the variables $p_0, p_1, \ldots$ and connectives $\Box, \bot$, where $\Box$ is unary and $\bot$ is a constant.

Define the following sequence of rules:

$$R_k = \{\frac{\Box^n \bot}{\Box^k \bot} : n \geq 0\} \; mboxforeach \; k \geq 0,$$

and a rule

$$R = \{\frac{\Box^n}{\Box^{n+1}} : n \geq 0\},$$

and put $C_k = C_R \cup R_k$.

LEMMA.

*(i) For all $k \geq 0$, $C_R$ and $C_k$ are structural and finite consequences and therefore regular.*

*(ii)* $C_{k+1} < C_k$, *all* $k \geq 0$

*(iii)* $inf\{C_k : k \geq 0\} = C_R$

Theorem 1.

*(i) The degree of complexity and the degree of uniformity of $C_k$ equal $k+2$, all $k \geq 0$.*

*(ii) The degree of complexity and the degree of uniformity of $C_R$ equal $\aleph_0$.*

Let $I$ be the identity consequence on $L$, i.e. $I(X) = X$, all $X \subseteq L$.

Theorem 2. *The logic $I$ has the degree of uniformity equal to $2^{\aleph_0}$ and the degree of complexity equal to $\aleph_0$.*

## References

[1] J. Hawranek, J. Zygmunt, *A theorem on the degree of complexity of some sentential logics*, this **Bulletin**, pp. 67–71.

[2] R. Wójcicki, *Some remarks on the consequence operation in sentential logics*, **Fundamenta Mathematicae** 68 (1970), pp. 269–279.

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