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SOME REMARKS ON BCK-ALGEBRAS

This is an abstract of the paper presented at the seminar held by prof. A. Wroński at the Jagiellonian University.

It was announced in [1] that T. Traczyk showed that any commutative BCK-algebra such that any two elements have an upper bound is a distributive lattice. In this paper we give a generalization of that theorem.

Let us recall a few fundamental definitions.

By a BCK-algebra we mean a general algebra $\langle X, *, 0\rangle$ of type $\langle 2, 0\rangle$ satisfying the following axioms:

(1) $(x * y) * (x * z) \leq z * y$,
(2) $x * (x * y) \leq y$,
(3) $x \leq x$,
(4) $0 \leq x$,
(5) $x \leq y$ and $y \leq x$ imply $x = y$,

where $x \leq y$ means $x * y = 0$.

It was proved in [2] that $\leq$ is a partial ordering. So $\langle X, \leq\rangle$ is a poset.

A BCK-algebra X is called commutative if for any $x, y \in X$

$$x * (x * y) = y * (y * x)$$

A BCK-algebra such that any two elements have an upper bound, in the sense of $\leq$, is called directed.

First let us note the following key Lemma.

Lemma. *In a commutative directed BCK-algebra*

$$(y * x) * (x * y) = y * x$$

Following [3], by a Łukasiewicz algebra we mean a general algebra $\langle Y, *, 0\rangle$ of type $\langle 2, 0\rangle$ satisfying the following axioms:

(6) $x = x * 0$,
(7) $x * y \leq x$,
(8) $(x * y) * (x * z) \leq z * y$,
(9) $X * (x * y) = y * (y * x)$,
(10) $x * y = (x * y) * (y * x)$.

It was proved in [2] that (6) and (7) hold in every BCK-algebra, so using Lemma we obtain

Corollary. *A commutative directed BCK-algebra is a Łukasiewicz algebra.*

Using Lemma we can prove the following theorem which gives us an equational characterization of subdirectly irreducible, linearly ordered BCK-algebras.

Theorem 1. *In a subdricetly irreducible BCK-algebra X the following conditions are equivalent:*

a) X *is linearly ordered by* $\leq$*;*
b) $(x * (x * (y * z))) * (x * (zXy)) = 0$ *holds* $j = X$.

The following theorem is a generalization of the result obtained by T. Traczyk.

Theorem 2. *A commutative directed BCK-algebra is a subdirect product of linearly ordered BCK-algebras.*

References

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[3] Y. Komori, *The separation theorem of the $\aleph_0$-valued Łukasiewicz propositional logic*, **reports of Faculty of Science**, Shizuoka University

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