Bulletin of the **S**ection of **L**ogic
Volume 9/3 (1980), pp. 122–123

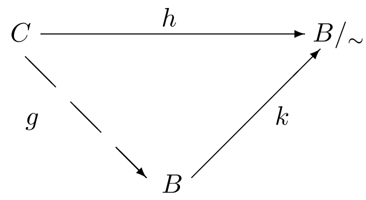
reedition 2010 [original edition, pp. 122–124]

Jacek Hawranek

A MATRIX ADEQUATE FOR $S5$ WITH MP AND RN

This is an abstract of a lecture read at the seminar of the Section of Logic, Polish Academy of Sciences, Wrocław, January 1980.

Definition. A Boolean algebra C is projective if every homomorphism from C to a quotient can be lifted to the numerator, i.e., the following diagram



commutes for an arbitrary Boolean algebra B, any congruence $\sim$ on B, and any homomorphism h.

Lemma (cf. [1]). *Each countable Boolean algebra is projective.*

Corollary. *Each countable Boolean algebra B is a retract of the countable free Boolean algebra B_0, i.e., there are homomorphisms g, f such that*

$$B \xrightarrow[g]{1-1} B_0 \xrightarrow[f]{onto} B$$

and $f \circ g = id$.

Let H_0 be the Henle algebra resulting from B_0 by adding a new unary operator I defined as follows: $I1 = 1$ and $Ia = 0$ if $a \neq 1$. R. Suszko (unpublished) has proved that the matrix (H_0, U_0), where U_0 is a special ultrafilter in B_0, is strongly adequate for the consequence based on $S5$ and MP (modus ponens).

Using the same method we shall show that the matrix $(H_0, \{1\})$ is strongly adequate for $S5$ with MP and RN (the rule of necessitation). Denote this consequence by Cn.

LEMMA. *Let H be a countable Henle algebra. Then the matrix $(H, \{1\})$ may be embedded into $(B_0, \{1\})$.*

THEOREM. *The class of all countable Henle algebras with the unit as the designated element is strongly adequate for Cn.*

In the proof we use the following facts:

(1) The deduction theorem for Cn holds in the form:

$$B \in Cn(Z, \{A\}) \text{ iff } \Box A \to B \in Cn(Z)$$

(2) The family of all Cn-consistent sets is a basis for Cn.

(3) Each Cn-maximal set Z has the property:

$$\Box A \in Z \text{ or } \neg\Box A \in Z$$

for every formula A.

Now observe that $(H_0, \{1\})$ belongs to the class $Matr(Cn) = \{\mathfrak{M} : Cn \leq Cn_{\mathfrak{M}}\}$ and by the latter Lemma we get

THEOREM. *The matrix $(H_0, \{1\})$ is strongly adequate for Cn.*

References

[1] P. R. Halmos, **Lectures on Boolean algebras**, Princeton 1963.

[2] H. Rasiowa, **An algebraic approach to non-classical logics**, Warsaw 1974.

[3] R. Suszko, *Identity connective and modality*, **Studia Logica** 27 (1971), pp. 7–42.

The Section of Logic
Institute of Philosophy and Sociology
Polish Academy of Sciences