Bulletin of the **S**ection of **L**ogic
Volume 9/4 (1980), pp. 159–161

reedition 2010 [original edition, pp. 159–162]

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A NOTE ON QUANTIFIED SIGNIFICANCE LOGICS

In 7.5 of [3] Goddard and Routly propose two logical systems of predicate calculus each encorporating certain significance restrictions.

In the first $PM1$, they read, in this connection '$\vdash A$' as 'the significance restriction of A is a theorem', where those variables in A that have free occurrences in A are restricted to their appropriate significance ranges. Using this interpretation of $\vdash$ they then use the usual form of modus ponens:

$\underline{R1}$ $$\vdash A,\ \vdash A \supset B \rightarrow \vdash B,$$

which clearly fails to hold here.

Consider for example there case where A and B both contain the free variable u and that A is significant if $u \in X_A$ (the significance range of A) while B is significant if $u \in X_B$. Then $A \supset B$ is significant if $u \in X_A \cap X_B$ and $R1$ will only give us $\vdash B$ for $u \in X_A \cap X_B$.

Another interpretation of '$\vdash A$' namely: 'if A is derivable independently of significance considerations, then A is a thesis for its significant values only', also does not seem satisfactory as it leaves open the possibility of having a particular substitution instance of A derivable but not a thesis. Even if SA (A is significant) could be derived for this A, SA may also be a nonthesis.

In a second system $PM2$, they replace $\vdash$ be $\in$, where '$\in A$'reads the universal closure of A is a theorem (where of course, each universal quantifier ranges of the significance range of the appropriate free variable).

Their modus ponens now becomes:

$\underline{R1}$ $$\in A,\ \in A \supset B \rightarrow \in B.$$

Taking the one variable case again, $\in A$ means A holds for all $u \in X_A$, $\in A \supset B$ means that $A \supset B$ holds for all $u \in X_A \cap X_B$ so that we have B for $u \in X_A \cap X_B$ rather than X_B.

If in this system, or even some part of it a given variable u has a range X_u over which all the predicate involving u in the (part) system are significant, this problem can be resolved. '$\vdash A$' can then be read as 'A with its free variables $u_1, u_2, \ldots, u_n$ restricted to $X_{u_1}, X_{u_2}, \ldots, X_{u_n}$ is a theorem'. Of course in general X_{u_1} will only be a subset of the significance range of u_1 in any particular formula.

An alternative solution involves retaining $R1$ only for cases where A and B have no free variables and using, for the one free variable case, the restricted generality of [2]. Writing $A(u)$ and $B(u)$ for A and B of $R1$ we then have:

$$\vdash A(V),\ \vdash A(u) \supset_u B(u) \rightarrow \vdash B(V)$$

where '$A(u) \supset_u B(u)$' is read '$B(u)$ holds for all u for which $A(u)$ holds' and where V is a term for which $A(V)$ is derivable (as well as significant).

We could in fact assume:

$$D \vdash S(D)$$

i.e. whenever D is derivable, D is significant.

In the case of several free variables $u_1, u_2, \ldots, u_n$, the generalized restricted generality of [1] can be used and we have instead of $R1$:

$$\vdash A(V_1, V_2, \ldots, V_n), \vdash A(u_1, u_2, \ldots, u_n) \supset u_1, u_2, \ldots, u_n^{B(u_1, u_2, \ldots, u_n)}$$

$$\rightarrow \vdash B(V_1, V_2, \ldots, V_n).$$

Of the quantifiers proposed in [3], those restricted to significance ranges can also be conveniently represented using restricted generality.

$(u)A(u)$, where u is understood to range only over those values for which $A(u)$ is significant, we can in fact define by $S(A(u)) \supset_u A(u)$.

Generalization ($R2$ in [3]) then becomes:

$$S(A(u)) \vdash A(u) \rightarrow (u)A(u)$$

(where the $\vdash$ is now interpreted in the usual fashion), and the remaining axioms and false of $PM1$ could become:

$P1$ If A is a substitution instance of an S_0-tautology then $SA \vdash A$.
$P2$ $SA, S((u)B) \vdash (u)(A \vee B) \supset .A \vee (u)B$.
$P3$ $S(A(V)) \vdash (u)A(u) \supset A(V)$.

Clearly the Behmann formulae such as

$$(x)f(x)\ ((x)f(x) \supset g(r)) \supset (x)g(x)$$

are not provable under this definition of (x), which is as required.

References

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[3] L. Goddard and R. Routley, **The Logic of Significance and Context**, Scottish Academic Press, Edinburgh 1973.

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