Prospects for classical theory of definite descriptions

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Application to DD:

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The general theory of term-forming operators and iota-operator:

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How does it work for DD?

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However, it can be argued that Russellian approach is not classical.

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The first extension is incomplete and the second is equivalent to Fregean theory of DD (the chosen object theory) and in fact redundant.

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HA $t = \imath x\varphi(x) \leftrightarrow \forall x(\varphi(x) \leftrightarrow x = t)$
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HA (LA) implies RAE'.

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LA $\forall y(y = \imath x\varphi(x) \leftrightarrow \forall x(\varphi(x) \leftrightarrow x = y)$

$HA^{\leftarrow}$ implies $RA^{\leftarrow}$.
In case on NFFOLI HA implies RA.

Application to DD:

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RoA $\exists_1 x\varphi(x) \rightarrow \forall x(x = \iota x\varphi(x) \leftrightarrow \varphi(x))$
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SIAS $\varphi(\iota x\varphi(x))$
SIA $\exists_1 x\varphi(x) \rightarrow \varphi(\iota x\varphi(x))$ or PD
SIAW $\iota y(y = \iota x\varphi(x)) = \iota x\varphi(x)$

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RoA, RoAW, KMA, SIA are equivalent.
SIAS implies SIA
SIA implies SIAW.
$RA^{\leftarrow}$ implies SIA (RoA, RoAW, KMA).

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HA $t = \imath x\varphi(x) \leftrightarrow \forall x(\varphi(x) \leftrightarrow x = t)$
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HA implies RoA.

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HA implies RoA.
In particular $HA^{\rightarrow}$ implies RoAW (so in fact RoA, KMA, SIA).

Application to DD:

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They are not independent:

DCA $\neg\exists_1 x\varphi(x) \rightarrow \iota x\varphi(x) = \iota x(x \neq x)$ or ID
SIA $\exists_1 x\varphi(x) \rightarrow \varphi(\iota x\varphi(x))$ or PD

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They are not independent:

DCA $\neg\exists_1 x\varphi(x) \to \iota x\varphi(x) = \iota x(x \neq x)$ or ID
SIA $\exists_1 x\varphi(x) \to \varphi(\iota x\varphi(x))$ or PD
FA $\psi(\imath x\varphi(x)) \leftrightarrow \exists y(\forall x(\varphi(x) \leftrightarrow x = y) \wedge \psi(y)) \vee$
$(\neg\exists y\forall x(\varphi(x) \leftrightarrow x = y) \wedge \psi(\imath x(x \neq x)))$

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FA $\psi(\imath x\varphi(x)) \ \leftrightarrow\ \exists y(\forall x(\varphi(x) \leftrightarrow x = y) \land \psi(y))\lor$
$(\neg\exists y\forall x(\varphi(x) \leftrightarrow x = y) \land \psi(\imath x(x \neq x)))$

FA is equivalent to DCA + SIA (KMA, RoA, RoAW).

Application to DD:

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1. $A(\imath x(Bx \wedge \neg Bx)) \vee \neg A(\imath x(Bx \wedge \neg Bx)) \rightarrow \exists y(\forall x(Bx \wedge \neg Bx \leftrightarrow x = y) \wedge (Ay \vee \neg Ay))$
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3. $\exists y(\forall x(Bx \wedge \neg Bx \leftrightarrow x = y) \wedge (Ay \vee \neg Ay))$ 1, 2

4.1. $\forall x(Bx \wedge \neg Bx \leftrightarrow x = a) \wedge (Aa \vee \neg Aa)$
4.2. $\forall x(Bx \wedge \neg Bx \leftrightarrow x = a)$
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The way out: restrict RA (or at least $RA^{\rightarrow}$) to atomic formulae. Note that in this case it does not hold and that RA implies RAE' and RA implies RAE only if we treat E as atomic predicate.

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In fact the last line shows that SIAS is also unacceptable (it is its instance).

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In fact the last line shows that SIAS is also unacceptable (it is its instance). Similarly RAE leads to contradiction if φ is $Ax \wedge \neg Ax$.
The way out: restrict HA to $HA^{\leftarrow}$ or to t a variable (at least for $HA^{\rightarrow}$) or, in case E is a primitive predicate, use:
HAE $Et \rightarrow (t = \imath x\varphi(x) \leftrightarrow \forall x(\varphi(x) \leftrightarrow x = t))$
not equivalent to LA in CFOLI (although equivalent to LA in FFOLI).

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3. Russell's Existence principle RAE leads to a paradox.

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1.	$\exists_1 y(y = \iota x \varphi(x))$	CFOLI thesis
2.	$E \iota y(y = \iota x \varphi(x))$	1, $RAE^{\leftarrow}$
3.	$\iota y(y = \iota x \varphi(x)) = \iota x \varphi(x)$	SIAW
4.	$E \iota x \varphi(x)$	2, 3, LL
5.	$\exists_1 x \varphi(x)$	4, $RAE^{\rightarrow}$

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or

$RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

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or equivalently FA.

$$\psi(\imath x\varphi(x)) \leftrightarrow \exists y(\forall x(\varphi(x) \leftrightarrow x = y) \wedge \psi(y)) \vee (\neg\exists y\forall x(\varphi(x) \leftrightarrow x = y) \wedge \psi(\imath x(x \neq x)))$$

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which gives a Fregean logic with the chosen object; let's use a special constant i for it instead of $\imath x(x \neq x)$.

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Next choice: either E is 1) definable (RAE') or 2) primitive (RAE).

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1) we have no choice but to reject $RAE'^{\rightarrow}$ since

$RAE'^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow \exists y(y = \imath x\varphi(x))$

is a thesis of CFOLI.

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In consequence we must reject $RA^{\rightarrow}$ or restrict it to atoms (which is desirable anyway, see above). We must also reject HA (LA) or at least $HA^{\rightarrow}$ ($LA^{\rightarrow}$), even if t is restricted to variables.

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2. Meinongian approach

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In consequence we must reject $RA^{\rightarrow}$ or restrict it to atoms (which is desirable anyway, see above). We must also reject HA (LA) or at least $HA^{\rightarrow}$ ($LA^{\rightarrow}$), even if t is restricted to variables.

2) we can consider both options and have two possibilities:

1. Fregean approach
2. Meinongian approach

The first accepts $RAE^{\leftarrow}$, the second $RAE^{\rightarrow}$.

Application to DD:

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Ways out (in CFOLI) - option 2:

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Since it holds that $\forall x(Ex \vee \neg Ex)$ we have in particular $Ei \vee \neg Ei$

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2a) accept $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

Application to DD:

Ways out (in CFOLI) - option 2:

Since it holds that $\forall x(Ex \lor \neg Ex)$ we have in particular $Ei \lor \neg Ei$

2a) accept $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

(which is not longer derivable like $RAE'^{\leftarrow}$).

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2a) accept $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

(which is not longer derivable like $RAE'^{\leftarrow}$).

We have: $RAE^{\leftarrow}$ is equivalent to $\forall xEx$ and implies Ei

Application to DD:

Ways out (in CFOLI) - option 2:

Since it holds that $\forall x(Ex \lor \neg Ex)$ we have in particular $Ei \lor \neg Ei$

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We have: $RAE^{\leftarrow}$ is equivalent to $\forall xEx$ and implies Ei
Hence $\forall xEx$ is a thesis and moreover it implies that Ei is a thesis too.

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Ways out (in CFOLI) - option 2:

Since it holds that $\forall x(Ex \vee \neg Ex)$ we have in particular $Ei \vee \neg Ei$

2a) accept $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

(which is not longer derivable like $RAE'^{\leftarrow}$).

We have: $RAE^{\leftarrow}$ is equivalent to $\forall xEx$ and implies Ei
Hence $\forall xEx$ is a thesis and moreover it implies that Ei is a thesis too.
In fact it leads to rejection of $RAE^{\rightarrow}$ by the following counterexample: $E\imath x(x \neq x)$ and $\neg\exists_1 x(x \neq x)$.

Application to DD:

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Ways out (in CFOLI) - option 2:

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2b) accept $RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

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Ways out (in CFOLI) - option 2:

2b) accept $RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

It holds that it is equivalent to $\neg Ei$.

Application to DD:

Ways out (in CFOLI) - option 2:

2b) accept $RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

It holds that it is equivalent to $\neg Ei$.

Note that $\neg Ei$ implies $\exists x \neg Ex$. Hence option 2b introduces a kind of Meinongianism; there are objects (in the domain of quantification) which do not exist; at least one such object – i.

Application to DD:

Ways out (in CFOLI) - option 2:

2b) accept $RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

It holds that it is equivalent to $\neg Ei$.

Note that $\neg Ei$ implies $\exists x \neg Ex$. Hence option 2b introduces a kind of Meinongianism; there are objects (in the domain of quantification) which do not exist; at least one such object – i.

Moreover, this option leads to rejection of $RAE^{\leftarrow}$ by the following counterexample: $\exists_1 y(y = i)$ and $\neg E\imath y(y = i)$.

Alternative formalisation:

Formal account of Fregean theory:

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In the framework of Tableaux a system of Bencivenga, Lambert and van Fraasen:

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Formal account of Fregean theory:

In the framework of Tableaux a system of Bencivenga, Lambert and van Fraasen:

$$\frac{\exists y(\forall x(\varphi(x) \leftrightarrow x = y) \land \psi(y)), \Gamma \Rightarrow \Delta \qquad \neg\exists y \forall x(\varphi(x) \leftrightarrow x = y) \land \psi(\imath xx \neq x), \Gamma \Rightarrow \Delta}{\psi(\imath x\varphi), \Gamma \Rightarrow \Delta}$$

where $\psi(\imath x\varphi)$ is a predicate or negated predicate with $\imath x\varphi$ as an argument.

The Fregean Approach:

Kalish and Montague formalisation:

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Kalish and Montague formalisation:

Two rules based on PD (SIA) and ID (DCA) added to ND-CFOLI:

$\exists y \forall x(\varphi(x) \leftrightarrow x = y) \vdash \varphi(\imath x \varphi)$
$\neg\exists y \forall x(\varphi(x) \leftrightarrow x = y) \vdash \imath x \varphi = i$

where i is a name of fixed denotation for all improper DD.

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FA (as well as EXT and AV) are derivable.

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FA (as well as EXT and AV) are derivable.

Alternatively (and equivalently) they characterised the Freagean system axiomatically by means of:

$$\forall y(\forall x(\varphi(x) \leftrightarrow x = y) \rightarrow \imath x \varphi = y)$$
$$\neg \exists y \forall x(\varphi(x) \leftrightarrow x = y) \rightarrow \imath x \varphi = i$$

SC for KM

Rules for identity:

SC for KM

Rules for identity:

SC for CFOL +

$$(=\Rightarrow)\ \frac{t = t, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}$$

$$(\Rightarrow=)\ \frac{\Gamma \Rightarrow \Delta, \varphi[x/t_1] \qquad \Pi \Rightarrow \Sigma, t_1 = t_2}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, \varphi[x/t_2]}$$

for φ atomic

Rules for descriptions:

Rules for descriptions:

$(\Rightarrow \imath)^1 \quad \dfrac{\varphi[x/a], \Gamma \Rightarrow \Delta, t = a \qquad t = a, \Pi \Rightarrow \Sigma, \varphi[x/a]}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, t = \imath x\varphi}$

1. where a is not in $\Gamma, \Delta, \Pi, \Sigma, \varphi$ and t is not i.

$(\Rightarrow i1)^2 \quad \dfrac{\varphi[x/a], \Gamma \Rightarrow \Delta,}{\Gamma \Rightarrow \Delta, i = \imath x\varphi}$

2. where a is not in Γ, Δ, φ.

$$(\Rightarrow i2)\ \frac{\Gamma \Rightarrow \Delta, \varphi[x/t_1] \qquad \Gamma_2 \Rightarrow \Delta_2, \varphi[x/t_2] \qquad t_1 = t_2, \Pi \Rightarrow \Sigma}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, i = \imath x\varphi}$$

Equivalence with KM:

SC for KM

Equivalence with KM:

Theorem

If $\Gamma \vdash \varphi$ *in KM system, then* $\vdash \Gamma \Rightarrow \varphi$.

Equivalence with KM:

Theorem

If $\Gamma \vdash \varphi$ *in KM system, then* $\vdash \Gamma \Rightarrow \varphi$*.*

and

Theorem

If $\vdash \Gamma \Rightarrow \Delta$*, then* $\Gamma \vdash \vee\Delta$ *in KM system.*

Equivalence with KM:

Equivalence with KM:

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\varphi(b) \Rightarrow \varphi(b)}{\varphi(b) \Rightarrow \underline{\varphi(b), a = b}}\,(W)
\quad
\dfrac{a = b \Rightarrow a = b}{\underline{\varphi(b)}, a = \underline{b \Rightarrow} a = b}\,(W)
}{\underline{\varphi(b) \leftrightarrow a = b}, \varphi(b) \Rightarrow a = b}\,(\leftrightarrow\Rightarrow)
}{\forall x(\varphi(x) \leftrightarrow a = x), \underline{\varphi(b)} \Rightarrow \underline{a = b}}\,(\forall\Rightarrow)
\qquad
\dfrac{
\dfrac{
\dfrac{a = b \Rightarrow a = b}{a = b \Rightarrow \underline{\varphi(b), a = b}}\,(W)
\quad
\dfrac{\varphi(b) \Rightarrow \varphi(b)}{\underline{\varphi(b), a = b} \Rightarrow \varphi(b)}\,(W)
}{\underline{\varphi(b) \leftrightarrow a = b}, a = b \Rightarrow \varphi(b)}\,(\leftrightarrow\Rightarrow)
}{\forall x(\varphi(x) \leftrightarrow a = x), \underline{a = b} \Rightarrow \underline{\varphi(b)}}\,(\forall\Rightarrow)
}{
\dfrac{
\dfrac{\forall x(\varphi(x) \leftrightarrow a = x) \Rightarrow a = \imath x\varphi(x)}{\Rightarrow \forall x(\varphi(x) \leftrightarrow a = x) \rightarrow a = \imath x\varphi(x)}\,(\Rightarrow\rightarrow)
}{\Rightarrow \forall y(\forall x(\varphi(x) \leftrightarrow y = x) \rightarrow y = \imath x\varphi(x))}\,(\Rightarrow\forall)
}\,(\Rightarrow \imath)
$$

where a is new.

SC for KM

Equivalence with KM:

Equivalence with KM:

$$(\Rightarrow i2)\ \frac{\varphi(a) \Rightarrow \varphi(a) \qquad \varphi(b) \Rightarrow \varphi(b) \qquad a = b \Rightarrow a = b}{\varphi(a), \varphi(b) \Rightarrow a = b, i = \imath x\varphi(x)}$$

$$(\Rightarrow\leftrightarrow)\ \frac{\varphi(a), \varphi(b) \Rightarrow a = b, i = \imath x\varphi(x) \qquad \varphi(a), a = b \Rightarrow \varphi(b)}{\varphi(a) \Rightarrow \varphi(b) \leftrightarrow a = b, i = \imath x\varphi(x)}$$

$$(\Rightarrow \forall)\ \frac{\varphi(a) \Rightarrow \varphi(b) \leftrightarrow a = b, i = \imath x\varphi(x)}{\varphi(a) \Rightarrow \forall x(\varphi(x) \leftrightarrow a = x), i = \imath x\varphi(x)}$$

$$(\Rightarrow \exists)\ \frac{\varphi(a) \Rightarrow \forall x(\varphi(x) \leftrightarrow a = x), i = \imath x\varphi(x)}{\varphi(a) \Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x)}$$

$$(\Rightarrow i1)\ \frac{\varphi(a) \Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x)}{\Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x), i = \imath x\varphi(x)}$$

$$(\Rightarrow C)\ \frac{\Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x), i = \imath x\varphi(x)}{\Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x)}$$

$$(\neg \Rightarrow)\ \frac{\Rightarrow \exists y\forall x(\varphi(x) \leftrightarrow y = x), i = \imath x\varphi(x)}{\neg\exists y\forall x(\varphi(x) \leftrightarrow y = x) \Rightarrow i = \imath x\varphi(x)}$$

$$(\Rightarrow\rightarrow)\ \frac{\neg\exists y\forall x(\varphi(x) \leftrightarrow y = x) \Rightarrow i = \imath x\varphi(x)}{\Rightarrow \neg\exists y\forall x(\varphi(x) \leftrightarrow y = x) \rightarrow i = \imath x\varphi(x)}$$

where a and b are new; rightmost sequent by $(\Rightarrow=)$

Equivalence with KM:

Equivalence with KM:

Derivability of $(\Rightarrow \imath)$:

$$\dfrac{\dfrac{\dfrac{\underline{\varphi(a)}, \Gamma \Rightarrow \Delta, \underline{t = a} \qquad \underline{t = a}, \Pi \Rightarrow \Sigma, \underline{\varphi(a)}}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, \underline{\varphi(a) \leftrightarrow t = a}}\,(\Rightarrow\leftrightarrow)}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, \underline{\forall x(\varphi(x) \leftrightarrow t = x)}}\,(\Rightarrow \forall) \qquad \underline{\forall x(\varphi(x) \leftrightarrow t = x)} \Rightarrow t = \imath x\varphi(x)}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, t = \imath x\varphi(x)}\,(Cut)$$

where a is new.

SC for KM

Equivalence with KM:

Equivalence with KM:

$(\Rightarrow i1)$:

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{a = a \Rightarrow a = a}{a = a \Rightarrow a = a, \varphi(a)}(W)
\qquad
\dfrac{\varphi(a), \Gamma \Rightarrow \Delta}{\varphi(a), a = a, \Gamma \Rightarrow \Delta}
}{\varphi(a) \leftrightarrow a = a, \Gamma \Rightarrow \Delta}(\leftrightarrow\Rightarrow)
}{\forall x(\varphi(x) \leftrightarrow a = x), \Gamma \Rightarrow \Delta}(\forall \Rightarrow)
}{\exists y \forall x(\varphi(x) \leftrightarrow y = x), \Gamma \Rightarrow \Delta}(\exists \Rightarrow)
}{\Gamma \Rightarrow \Delta, \neg\exists y \forall x(\varphi(x) \leftrightarrow y = x)}(\Rightarrow \neg)
\qquad
\neg\exists y \forall x(\varphi(x) \leftrightarrow y = x) \Rightarrow i = \imath x \varphi(x)
}{\Gamma \Rightarrow \Delta, i = \imath x \varphi(x)}(Cut)
$$

where a is new.

SC for KM

Equivalence with KM:

Equivalence with KM:

For derivability of $(\Rightarrow i2)$ first prove:

$$
\cfrac{
  \cfrac{\Gamma \Rightarrow \Delta, \underline{\varphi(t_1)} \qquad \Pi \Rightarrow \Sigma, \underline{\varphi(t_2)}}{\Gamma, \Pi \Rightarrow \Delta, \Sigma, \underline{\varphi(t_1) \wedge \varphi(t_2)}}\,(\Rightarrow\wedge)
  \qquad
  \cfrac{\underline{t_1 = t_2}, \Lambda \Rightarrow \Theta}{\Lambda \Rightarrow \Theta, \underline{t_1 \neq t_2}}\,(\Rightarrow\neg)
}{
  \cfrac{\Gamma, \Pi, \Lambda \Rightarrow \Delta, \Sigma, \Theta, \underline{\varphi(t_1) \wedge \varphi(t_2) \wedge t_1 \neq t_2}}{\Gamma, \Pi, \Lambda \Rightarrow \Delta, \Sigma, \Theta, \exists x, y(\varphi(x) \wedge \varphi(y) \wedge x \neq y)}\,(\Rightarrow\exists)
}\,(\Rightarrow\wedge)
$$

Equivalence with KM:

SC for KM

Equivalence with KM:

$$\dfrac{\dfrac{\dfrac{\dfrac{\varphi(b)\Rightarrow\varphi(b)}{\varphi(b)\Rightarrow\underline{\varphi(b), b=c}}(\Rightarrow W)\qquad\dfrac{\dfrac{b=c\Rightarrow\underline{b=c}\qquad a=c\Rightarrow\underline{a=c}}{b=c, a=c\Rightarrow a=b}(\Rightarrow =)}{\underline{\varphi(b), b=c}, a=c\Rightarrow a=b}(W\Rightarrow)}{\varphi(b), a=c, \underline{\varphi(b)\leftrightarrow b=c}\Rightarrow a=b}(\leftrightarrow\Rightarrow)}{\varphi(b), a=c, \forall x(\varphi(x)\leftrightarrow x=c)\Rightarrow a=b}(\forall\Rightarrow)}{\varphi(a), a=c, \varphi(b), \forall x(\varphi(x)\leftrightarrow x=c)\Rightarrow a=b}(W\Rightarrow)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\varphi(a)\Rightarrow\varphi(a)}{\varphi(a)\Rightarrow\underline{\varphi(a), a=c}}(\Rightarrow W)\qquad\underline{\varphi(a), a=c}, \varphi(b), \forall x(\varphi(x)\leftrightarrow x=c)\Rightarrow a=b}{\varphi(a), \varphi(b), \underline{\varphi(a)\leftrightarrow a=c}, \forall x(\varphi(x)\leftrightarrow x=c)\Rightarrow a=b}(\leftrightarrow\Rightarrow)}{\varphi(a), \varphi(b), \underline{\forall x(\varphi(x)\leftrightarrow x=c)}, \forall x(\varphi(x)\leftrightarrow x=c)\Rightarrow a=b}(\forall\Rightarrow)}{\varphi(a), \varphi(b), \underline{\forall x(\varphi(x)\leftrightarrow x=c)}\Rightarrow a=b}(C\Rightarrow)}{\varphi(a), \varphi(b), \exists y\forall x(\varphi(x)\leftrightarrow x=y)\Rightarrow\underline{a=b}}(\exists\Rightarrow)}{\underline{\varphi(a), \varphi(b), a\neq b}, \exists y\forall x(\varphi(x)\leftrightarrow x=y)\Rightarrow}(\neg\Rightarrow)}{\underline{\varphi(a)\wedge\varphi(b)\wedge a\neq b}, \exists y\forall x(\varphi(x)\leftrightarrow x=y)\Rightarrow}(\wedge\Rightarrow)}{\exists x, y(\varphi(x)\wedge\varphi(y)\wedge x\neq y), \underline{\exists y\forall x(\varphi(x)\leftrightarrow x=y)}\Rightarrow}(\exists\Rightarrow)}{\exists x, y(\varphi(x)\wedge\varphi(y)\wedge x\neq y)\Rightarrow\neg\exists y\forall x(\varphi(x)\leftrightarrow x=y)}(\Rightarrow\neg)$$

where a, b, c new. The conclusion by cut with $\Gamma, \Pi, \Lambda\Rightarrow\Delta, \Sigma, \Theta, \exists x, y(\varphi(x)\wedge\varphi(y)\wedge x\neq y)$ and $\neg\exists y\forall x(\varphi(x)\leftrightarrow y=x)\Rightarrow i=\imath x\varphi(x)$.

The Fregean versus Meinongian Approach:

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KM system with added E:

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Fregean extension: $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

$$\frac{\varphi(a), \Gamma \Rightarrow \Delta, a = b \qquad a = b, \Gamma \Rightarrow \Delta, \varphi(a)}{\Gamma \Rightarrow \Delta, E\imath x\varphi(x)}$$

where a is not in Γ, Δ, φ

The Fregean versus Meinongian Approach:

KM system with added E:

Fregean extension: $RAE^{\leftarrow}$ $\exists_1 x\varphi(x) \rightarrow E\imath x\varphi(x)$

$$\frac{\varphi(a), \Gamma \Rightarrow \Delta, a = b \qquad a = b, \Gamma \Rightarrow \Delta, \varphi(a)}{\Gamma \Rightarrow \Delta, E\imath x\varphi(x)}$$

where a is not in Γ, Δ, φ

Meinongian extension: $RAE^{\rightarrow}$ $E\imath x\varphi(x) \rightarrow \exists_1 x\varphi(x)$

$$\frac{\varphi(a), \Gamma \Rightarrow \Delta}{E\imath x\varphi(x), \Gamma \Rightarrow \Delta}$$

where a is not in Γ, Δ, φ

$$\frac{\Gamma \Rightarrow \Delta, \varphi(b) \qquad \Gamma \Rightarrow \Delta, \varphi(c) \qquad b = c, \Gamma \Rightarrow \Delta}{E\imath x\varphi(x), \Gamma \Rightarrow \Delta}$$

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