Bisimulation for Propositional Modal Logic With Definite Descriptions

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Motivations

Definite descriptions are term-forming expressions, e.g., ‘the x such that $\varphi(x)$’.

Such expression have been intensively studied in first-order languages, but only recently considered *in propositional modal languages*.

In particular, we have introduced $\mathcal{ML}(\mathsf{DD})$ by adding operator $@_\varphi$ to modal logic:

- $@_\varphi \psi$ is to mean that *‘ψ holds in the modal world in which φ holds’*.

Motivations

Known results on the complexity of satisfiability checking:

- $\mathcal{H}(@)$-satisfiability is PSpace-*complete* (Areces, Blackburn, Marx),
- $\mathcal{MLC}$-satisfiability is ExpTime-*complete* with unary encoded numbers (PhD of Tobies),
- $\mathcal{MLC}$-satisfiability is NExpTime-*complete* with binary encoded numbers (Zawidzki, Schmidt, Tishkovsky).

We showed that:

- $\mathcal{ML}(\mathrm{DD})$-satisfiability is ExpTime-*complete*,
- $\mathcal{ML}(\mathrm{DD})$-satisfiability with Boolean DDs is PSpace-*complete*.

Motivations

We also studied *relative expressiveness* and showed the following results on equivalence preserving translations:

- $\mathcal{H}(@) \prec \mathcal{ML}(\mathrm{DD}) \prec \mathcal{MLC}$ (arbitrary frames)
- $\mathcal{H}(@) \prec_L \mathcal{ML}(\mathrm{DD}) \prec_L \mathcal{MLC}$ (linear frames)
- $\mathcal{H}(@) \prec_{\mathbb{Z}} \mathcal{ML}(\mathrm{DD}) \approx_{\mathbb{Z}} \mathcal{MLC}$ (integer frame)

It remains, however, unclear **what exactly does $\mathcal{ML}(\mathrm{DD})$ allow us to express**.

Contributions

Aiming to fill this gap we will provide *a bisimulation for* $\mathcal{ML}(\mathrm{DD})$.

Our $\mathcal{ML}(\mathrm{DD})$*-bisimulation enjoys*:

- the *bisimulation invariance property*, i.e.,
  bisimilar worlds satisfy the same $\mathcal{ML}(\mathrm{DD})$-formulas,
- the *Hennessy-Milner property*, i.e.,
  the opposite implication for image-finite (i.e., finite branching) models.

Logic $\mathcal{ML}(\mathsf{DD})$

Syntax of $\mathcal{ML}$(DD)

- We introduce operators $@_\varphi$, for any formula φ.
- $@_\varphi\psi$ is to mean that *'ψ holds in the unique world in which φ holds'*.

$\mathcal{ML}$(DD)-*formulas* are generated by

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \Diamond\varphi \mid @_\varphi\varphi,$$

We call $@_\varphi$ a *definite description*; we call it *Boolean* if so is φ.

Semantics of $\mathcal{ML}$(DD)

A *model* is a triple $\mathcal{M} = (W, R, V)$ where:

- $W \neq \emptyset$,
- $R \subseteq W \times W$,
- V : PROP $\longrightarrow \mathcal{P}(W)$.

Satisfaction of a formula in $\mathcal{M}$ and $w \in W$ is defined recursively:

$\mathcal{M}, w \models p$	iff	$w \in V(p)$, for each $p \in$ PROP
$\mathcal{M}, w \models \neg\varphi$	iff	$\mathcal{M}, w \not\models \varphi$
$\mathcal{M}, w \models \varphi \vee \psi$	iff	$\mathcal{M}, w \models \varphi$ or $\mathcal{M}, w \models \psi$
$\mathcal{M}, w \models \Diamond\varphi$	iff	there exists $v \in W$ such that $(w, v) \in R$ and $\mathcal{M}, v \models \varphi$
$\mathcal{M}, w \models @_\varphi\psi$	iff	there exists $v \in W$ such that $\mathcal{M}, v \models \varphi$ and $\mathcal{M}, v \models \psi$ and $\mathcal{M}, v' \not\models \varphi$ for all $v' \neq v$ in W

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5. $@_p \Diamond p$ 'there exists exactly one world which satisfies p; moreover this world can be accessed from itself',
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7. $@_{(p\vee\neg\varphi)}(\varphi)$ 'formula φ holds in every world (and p holds in exactly one world)',
8. $@_p\varphi$ 'formula φ holds in some world (and p holds in exactly one world and this world is one of the worlds in which φ holds)'.

Bisimulations

Standard bisimulation

Definition. An *ML-bisimulation* between $\mathcal{M} = (W, R, V)$ and $\mathcal{M}' = (W', R', V')$ is any $Z \subseteq W \times W'$ such that if $(w, w') \in Z$:

Atom: w and w' satisfy the same atoms,

Zig: if there is $v \in W$ with $(w, v) \in R$, then there is $v' \in W'$ such that $(v, v') \in Z$ and $(w', v') \in R'$,

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Theorem (Bisimulation Invariance Lemma). If $\mathcal{M}, w \leftrightarrows_{\mathcal{ML}} \mathcal{M}', w'$ then w and w' satisfy the same $\mathcal{ML}$-formulas.

Theorem (Hennessy-Milner Theorem). Assume that $\mathcal{M}$ and $\mathcal{M}'$ are image-finite. Then $\mathcal{M}, w \leftrightarrows_{\mathcal{ML}} \mathcal{M}', w'$ if and only if w and w' satisfy the same $\mathcal{ML}$-formulas.

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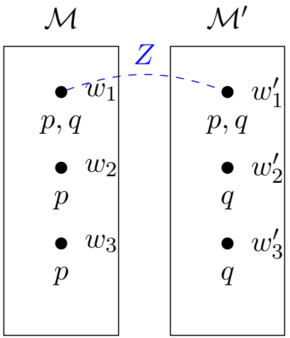

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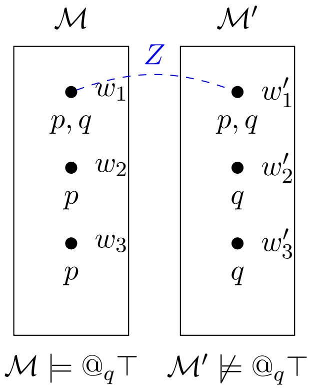

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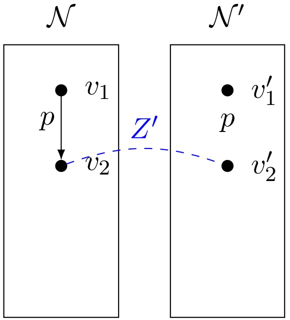

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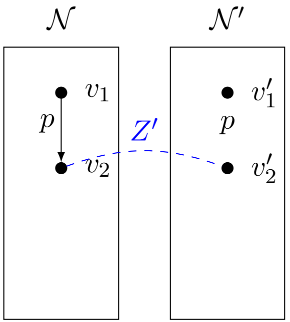

$\mathcal{N} \models @_p \Diamond \top$ $\quad \mathcal{N}' \not\models @_p \Diamond \top$

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Bisimulation for $\mathcal{ML}(\mathrm{DD})$

Names and named worlds

Definition. *Names*$(\mathcal{M})$ is the set of all $\mathcal{ML}$-formulas φ such that φ is satisfied in a unique world of $\mathcal{M}$.

Definition. *NamedWorlds*$(\mathcal{M})$ is the set of all worlds w such that $\mathcal{M}, w \models \varphi$, for some $\varphi \in \mathit{Names}(\mathcal{M})$.

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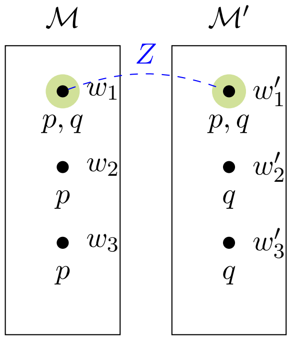

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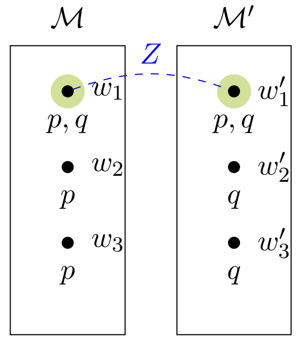

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Z does not satisfy Requirement 1:
$q \in$ *Names*($\mathcal{M}$), but $q \notin$ *Names*($\mathcal{M}'$).

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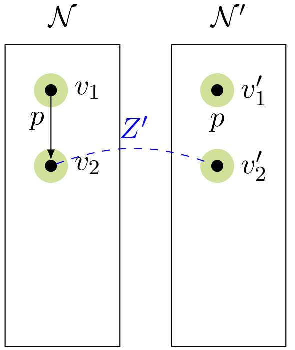

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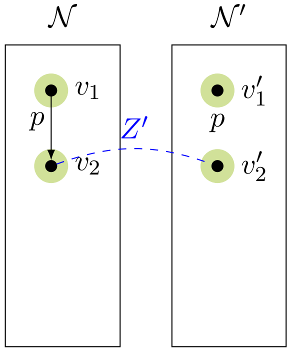

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Z' does not satisfy Requirement 2:
$v_1 \in$ *NamedWorlds*$(\mathcal{N})$ and
$v'_1 \in$ *NamedWorlds*$(\mathcal{N}')$,
but they are not related by Z'.

$\mathcal{ML}$(DD)-bisimulation

Definition. An $\mathcal{ML}$(DD)-*bisimulation* between $\mathcal{M}$ and $\mathcal{M}'$, with *Names*$(\mathcal{M})$ = *Names*$(\mathcal{M}')$, is any $\mathcal{ML}$-bisimulation Z such that:

Dom: the domain of Z contains *NamedWorlds*$(\mathcal{M})$,

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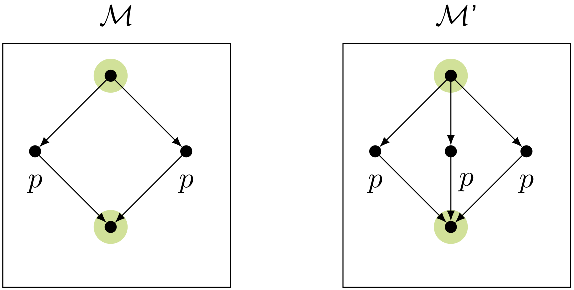

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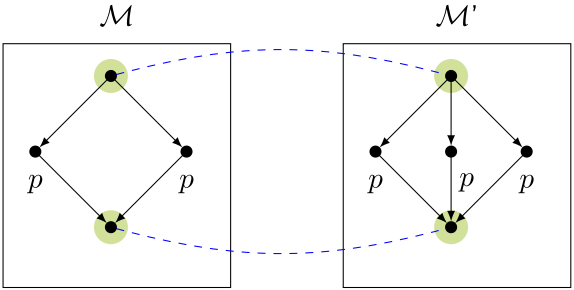

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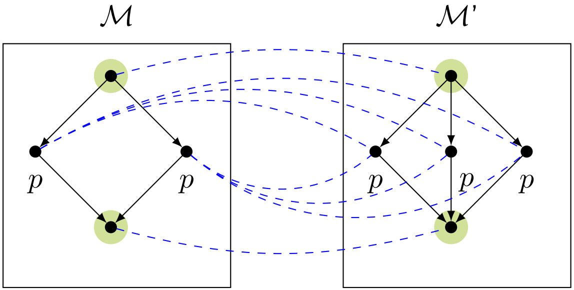

Properties of $\mathcal{ML}(\mathsf{DD})$-bisimulations

Basic properties

Proposition. If $\mathit{Names}(\mathcal{M}) \neq \mathit{Names}(\mathcal{M}')$, then there exists an $\mathcal{ML}$(DD)-formula φ such that $\mathcal{M} \models \varphi$ and $\mathcal{M}' \not\models \varphi$.

Indeed, if $\psi \in \mathit{Names}(\mathcal{M})$, but $\psi \notin \mathit{Names}(\mathcal{M}')$, then $\varphi = @_{\psi}\top$ witnesses proposition.

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Indeed, if $\psi \in \mathit{Names}(\mathcal{M})$, but $\psi \notin \mathit{Names}(\mathcal{M}')$, then $\varphi = @_\psi \top$ witnesses proposition.

Proposition. Let Z be an $\mathcal{ML}(\mathrm{DD})$-bisimulation between models $\mathcal{M} = (W, R, V)$ and $\mathcal{M}' = (W', R', V')$. Then $Z = Z_1 \cup Z_2$ where

- Z_1 is a bijection,
- $Z_1 \subseteq \mathit{NamedWorlds}(\mathcal{M}) \times \mathit{NamedWorlds}(M')$,
- $Z_2 \subseteq (W \setminus \mathit{NamedWorlds}(\mathcal{M})) \times (W' \setminus \mathit{NamedWorlds}(\mathcal{M}'))$.

Hence bisimilar models have the same number of named worlds, i.e., $|\mathit{NamedWorlds}(\mathcal{M})| = |\mathit{NamedWorlds}(\mathcal{M}')|$.

Removing nesting of $@$

Lemma. For each $\mathcal{ML}$(DD)-formula there exists an equivalent $\mathcal{ML}$(DD)-formula with no nesting of $@$.

For example $@_p @_{(@_q r)} s$ is equivalent to:

$$
\begin{array}{llllllll}
(@_q r & \wedge & @_\top s & \wedge & @_p \top) & \vee \\
(@_q r & \wedge & \neg @_\top s & \wedge & @_p \bot) & \vee \\
(\neg @_q r & \wedge & @_\top s & \wedge & @_p \top) & \vee \\
(\neg @_q r & \wedge & \neg @_\top s & \wedge & @_p \bot) & .
\end{array}
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Main results

Theorem (Bisimulation invariance property for $\mathcal{ML}(\mathsf{DD})$). If $\mathcal{M}, w \leftrightarrows_{\mathcal{ML}(\mathsf{DD})} \mathcal{M}', w'$ then w and w' satisfy the same $\mathcal{ML}(\mathsf{DD})$-formulas.

- Proof is by induction on formula structure.
- The interesting case is for a formula of the form $@_\varphi \psi$.

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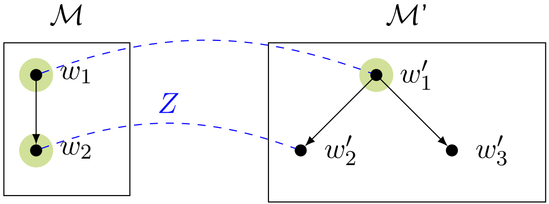
- Proof is by induction on formula structure.
- The interesting case is for a formula of the form $@_\varphi \psi$.

Theorem (Hennessy-Milner property for $\mathcal{ML}(\mathsf{DD})$). Assume that $\mathcal{M}$ and $\mathcal{M}'$ are image-finite, models. Then $\mathcal{M}, w \leftrightarroweq_{\mathcal{ML}(\mathsf{DD})} \mathcal{M}', w'$ if and only if w and w' satisfy the same $\mathcal{ML}(\mathsf{DD})$-formulas.

- Let $(w, w') \in Z$ if and only if w and w' satisfy the same $\mathcal{ML}(\mathsf{DD})$-formulas.
- We can show that Z is an $\mathcal{ML}(\mathsf{DD})$-bisimulation.
- The interesting part is to show that $\mathit{Names}(\mathcal{M}) = \mathit{Names}(\mathcal{M}')$ and that Z satisfies Dom and Rng.

Bisimulation for $\mathcal{ML}$(DD) with Boolean DDs



Z is not an $\mathcal{ML}$(DD)-bisimulation, but it should be a bisimulation if we allow for Boolean DDs only.

marks named worlds, nut not worlds have Boolean names

Definition.

- $\mathit{Names}_B(\mathcal{M}) = \{\varphi \in \mathit{Names}(\mathcal{M}) \mid \varphi \text{ is Boolean}\}$.
- $\mathit{NamedWorlds}_B(\mathcal{M}) = \{w \mid \mathcal{M}, w \models \varphi \text{ and } \varphi \in \mathit{Names}_B(\mathcal{M})\}$.

Definition. A $\mathcal{BML}$(DD)-bisimulation is defined as $\mathcal{ML}$(DD)-bisimulation but with *Names* and *NamedWorlds* replaced by Names_B and $\mathit{NamedWorlds}_B$, respectively.

$\mathcal{BML}$(DD)-bisimulation properties

Proposition. Each $\mathcal{ML}$(DD)-bisimulation is also an $\mathcal{BML}$(DD)-bisimulation, but not vice versa. Moreover each $\mathcal{BML}$(DD)-bisimulation is an $\mathcal{ML}$-bisimulation, but not vice versa.

Theorem (Bisimulation invariance property for $\mathcal{BML}$(DD)). If $\mathcal{M}, w \leftrightarroweq_{\mathcal{BML}(\mathsf{DD})} \mathcal{M}', w'$ then w and w' satisfy the same $\mathcal{BML}$(DD)-formulas.

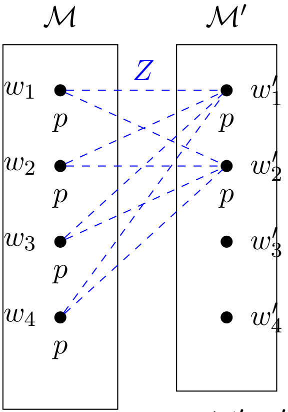
Theorem (Hennessy-Milner property for $\mathcal{BML}$(DD)). Assume that $\mathcal{M}$ and $\mathcal{M}'$ are image-finite, models. Then $\mathcal{M}, w \leftrightarroweq_{\mathcal{BML}(\mathsf{DD})} \mathcal{M}', w'$ if and only if w and w' satisfy the same $\mathcal{BML}$(DD)-formulas.

Applications of $\mathcal{ML}(\mathsf{DD})$-bisimulation

Non-definability of operators

In $\mathcal{ML}$(DD) (and in $\mathcal{BML}$(DD)) we cannot define:

- ‘everywhere’ (universal) operator A,
- the difference operator D,
- ‘somewhere’ operator E,
- counting operator $\exists_n$, for any $n \geq 2$.



$\mathcal{M}, w_1 \models \mathsf{A}p$
$\mathcal{M}, w_1 \not\models \mathsf{D}\neg p$
$\mathcal{M}, w_1 \not\models \mathsf{E}\neg p$

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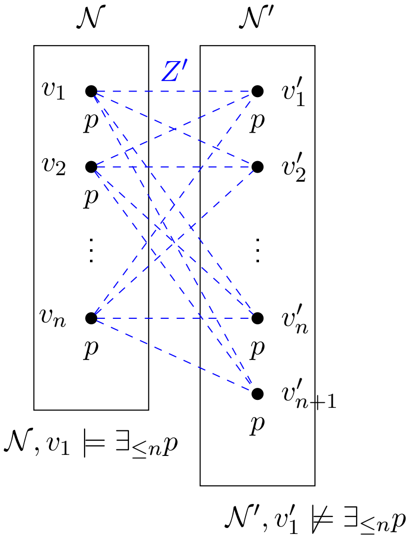

Conclusions

Conclusions and Future Work

$\mathcal{ML}(\mathrm{DD})$ extends modal logic with operators $@_\varphi$, where

- $@_\varphi \psi$ means that *'ψ holds in the world in which φ holds'*.

We defined an *$\mathcal{ML}(\mathrm{DD})$-bisimulation* which enjoys:

- the *bisimulation invariance property*,
- the *Hennessy-Milner property*.

Next steps:

- develop an algorithm constructing a (maximal) $\mathcal{ML}(\mathrm{DD})$-bisimulation between a pair of models.

Thank you for your attention

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