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Part I.

Invited Lectures

Current topics in Boolean connexive logic

Mateusz Klonowski

Nicolaus Copernicus University, Poland

mateusz.klonowski@umk.pl

Our paper will discuss the main topics of research in Boolean connexive logic (BCL) and some possible directions for further research development (for an introduction to BCL, see [5, 6]). We will try to consider the comments and suggestions of researchers interested in the broadly understood topic of connexive logics (for current trends in and an introduction to connexive logic, see [14, 15]). And that applies both to problems related to connexive logic in general (e.g., the problem of the origins of connexive logic, see [10, 9], cf. [13]) and to BCL in particular. The main topics of BCL research include:

1. philosophical motivations and applications of BCL (cf. [2, 5, 8, 11]),
2. connexive adaptation of philosophical relating logics (cf. [5, 8, 11, 16]),
3. proof theory for BCL (cf. [5, 6, 7]),
4. linguistic and semantic modification of the basic BCL: modal systems, systems of combined semantics, hyperconnexive systems, etc. (cf. [6, 7, 12, 16]),
5. comparison of BCL with other connexive logics (cf. [1, 12]).

Topics 2 and 3 are especially interesting to us, so our paper focuses on these two topics.

BCL might be considered a subfamily of relating logic (see [3, 4]), and topic 2 concerns the problem of modifications of philosophically motivated relating logics in such a way as to obtain connexive logic. Usually, such modification is related to an analysis of relationships of sentences due to, for instance, a relationship between the contents of sentences or a situation dependence. As part of the research on this topic, we examine to what extent a given relationship of sentences can constitute a special form of connexivity relation ("connection" or "coherence" relation, cf. [10]). This topic is, of course, strongly related to topics 1 and 4. If analyzing a given relationship between sentences leads to connexive logic, then such logic can be considered philosophically correct, i.e., well-motivated philosophically. Additionally, modifying a given philosophical logic usually requires modifying its semantic structure.

Topic 3 is interesting for us, as it is a natural supplement to topic 2 and topic 4. It is also related to topic 5 since an axiomatic or sequent presentation often makes comparing a given logic with other formal systems easier.

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How classical connexive logic can be?

Jacek Malinowski

Polish Academy o Sciences, Poland

studia.logica@gmail.com

In the articles [1, 2] we introduced the notions of Boolean connexive (BCL) and Boolean modal connexive logic (MBCL). We also defined the smallest systems of BCL and MBCL that satisfy the definition of connexive logic: they contain the principles of Aristotle, Boethius, and are closed under Modus Ponens [4].

The systems were defined according to the principle of Minimal Change Strategy (MCS), which states that as few changes as possible should be made. This is a version of Occam's principle, a widely accepted principle of economic formulation of scientific theories [3].

Therefore, although the implication behaves in these systems as a connexive implication, the remaining functors behave in a Boolean and therefore classical way. So we have made a small change to the classical logic to obtain connexive logics.

However, this change is still large, since we have rejected all classical laws for the implication of $\rightarrow$. Although we can define the material implication and all classical laws in our logics, the main implication of $\rightarrow$ is very weak: the set of principles of $\rightarrow$ is reduced to the connexive principles and what can be inferred from them by means of Modus Ponens.

The natural question then is, how do we extend the smallest BCL (or MBCL)? This is just a starting point for richer sets of formulas. Of course, one can add further counter-classical laws for $\rightarrow$. Obviously, there is the continuum of such extensions — as many as there are subsets of sets of formulas.

During one of the previous WCL we asked the question how to extend BCL to get as close as possible to classical logic (7th Workshop on Connexive Logics, 2022, UNAM, Mexico). Some extensions of that kind have been already proposed, for example in [5], [3].

The question then is how to extend the smallest BCL, getting closer to classical logic, i.e. making BCL more and more classical, but not to the point of trivializing it. This problem will be the central problem of this paper.

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Another Constructive Motivation for Some Connexive Logics

Satoru Niki

Ruhr University Bochum, Germany

Satoru.Niki@rub.de

H. Wansing's system **C** [5] is a connexive logic that has (positive) intuitionistic logic as its basis. As such, one natural way to motivate the logic would be to conceive it as an expansion of the Brouwerian system with a more constructive negation. If intuitionistic negation is retained, then this story leads to a slight variant of **C** with the falsity constant $\perp$; such a system has been investigated by D. Fazio and S.P. Odintsov [2] under the name $\mathbf{C}^{\perp}$.

A chief rival of $\mathbf{C}^{\perp}$ in this narrative would be the system $\mathbf{N4}^{\perp}$ [4] that expands the Almukdad-Nelson system **N4** [1] by the falsity constant. $\mathbf{C}^{\perp}$ and $\mathbf{N4}^{\perp}$ have different conditions for refuting an implication, but the classical-looking condition for $\mathbf{N4}^{\perp}$ might be seen as more intuitive and thus more favourable, than the condition for $\mathbf{C}^{\perp}$ that induces connexivity.

Against this picture, In this talk I will discuss a constructive criterion, according to which $\mathbf{C}^{\perp}$ can be a better candidate than $\mathbf{N4}^{\perp}$, when one seeks to augment the constructivity of intuitionistic logic. I will then examine the relationship between this constructive criterion and some extensions of $\mathbf{C}^{\perp}$.

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Non-classical conditionals and connexivity

Grigory Olkhovikov
Ruhr University Bochum, Germany
grigory.olkhovikov@rub.de

We begin by considering the following problem. Given a first-order logic QL and its propositional fragment L such that QL and L are subsystems of the classical first-order logic QCL and the classical propositional logic CL, respectively, how do we find a minimal normal (w.r.t. an appropriate relational semantics) conditional operator for this logic? One strategy would be to emulate the success of the classical conditional logic CK introduced in [1] and to look for its analogues on the basis of a given logic L. It often happens, however, that several different systems have a claim to provide such an analogue. We will argue for an approach (somewhat loosely inspired by [2]) where the ultimate touchstone for our choice in these cases is given by the standard translation *ST* of CK into QCL. Namely, the conditional extension LCK of L is the right analogue of CK on the basis of L iff *ST* embeds LCK into QL.

In our talk, we will show that this strategy works for both intuitionistic propositional logic IL and the paraconsistent variant of Nelson's logic of strong negation N4.

Turning next to the question of realizing connexive principles in a conditional logic extending a non-classical propositional basis L, we observe that the very nature of our criterion prevents the minimal conditional from displaying any connexive properties in case the implication of L fails to be connexive. Therefore, connexive conditional operators quite generally cannot be obtained as the minimal conditional operators on the basis of subsystems of CL. In case one is specifically interested in connexive conditionals, two strategies naturally suggest themselves: (1) one may try to realize connexive principles by extending LCK to some non-minimal conditional logic, and (2) one may look for analogues of CK on the basis of some logic L that has a connexive implication.

In both (1) and (2) one has to deal with additional challenges. As for (1), subsystems of classical logics often impose their own specific constraints on admissible extensions that are absent in CL; one example is Disjunction Property. As for (2), note that connexive logics are contra-classical and thus cannot be subsystems of CL. This means that our motivation for the choice of the right minimal conditional operator, which is based on the search for the correct analogue of the classical conditional logic CK must be put into a new perspective and at least somewhat generalized, which is not easy to do systematically, given the current state of research on conditional logics.

We will briefly assess the potential of following the strategy (1) in connection with CL, IL, and N4, and tentatively explore option (2) taking the connexive logic C, introduced in [3] as our main example.

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An interpretation of McCall's CC1

Francesco Paoli

University of Cagliari, Italy

paoli@unica.it

McCall's logic CC1 [3], one of the earliest and best known connexive systems in the literature, has been often criticised for possessing some implausible validities (and for lacking some plausible ones), as well as for the absence of a convincing intuitive interpretation. Inspired by some work by Herzberger [1] and Song et al. [4], we suggest a new account of CC1, aimed at vindicating its naturalness. Under this construal, sentences are characterised both by a *truth value* (true or false) and by a *content polarity* (positive or negative). In particular, in full accord with the ideas underlying many other connexive logics, a CC1 conditional is true if and only if 1) it comes out true as a material conditional; and 2) the contents of its antecedent and of its consequent are compatible, i.e., they have the same polarity. After showing that Angell and McCall's matrix for CC1 can be read along these lines, we prove the completeness of the system w.r.t. a class of algebraic models obtained via a certain construction on Boolean algebras. Finally, we discuss the prospects for a relating semantics in the style of [2].

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Part II.

Contributed Lectures

Queer feminist views on contradictory logics: A symbiotic relationship

Sara Ayhan

Ruhr University Bochum, Germany

sara.ayhan@rub.de

In this talk I want to investigate possible applications of queer feminist views[1] on (philosophy of) logic with respect to contradictory logics and especially bilateral representations of these. Thereby I want to show that, on the one hand, the formal set-up of contradictory logics makes them well-suited from the perspectives of feminist logic and, on the other hand, that queer feminist theories provide a relevant, and so far undeveloped, conceptual motivation for contradictory logics. Thus, applying contradictory logics to reasoning about queer feminist issues may prove fruitful both as a 'real-life' motivation for these rather marginalized logical systems and as a formal basis for a philosophical field that is still characterized by a distrust of formalism.

Feminist logic is an area of study that seems underrepresented both in logic and in feminist philosophy. The reason for the former is that many (most?) logicians simply do not see any connection or applicability of logic to feminist issues. The reason for the latter is that there is some feminist literature, e.g., [8], arguing that feminism and logic are in principle incompatible. However, proving both sides wrong in practice, feminist logic has developed as a small but upcoming discipline.[2] Feminist logic is often used as a short form including both what can be understood as 'feminist logic' and 'feminist philosophy of logic'. These two areas are very much intertwined, usually informing, affecting or guiding each other, and I do not see any necessity here to make a clear-cut distinction but to give some examples of what these can/do include:[3] Feminist logic (proper) may be conducted by formalizing notions that are especially important for feminist reasoning [12] and/or applications of logic(s) to feminist ends [11], while projects in feminist philosophy of logic usually go in a direction of devising, revising and/or arguing for logical systems from a distinctly feminist perspective. The latter has been done in [9], arguably the most central work in feminist logic, in which a feminist critique of classical logic is voiced. Plumwood sees classical logic as a "Logic of Domination" by implementing and perpetuating what she calls "du-

[1] Feminism can be broadly understood as the socio-political movement that aims to establish social, political, economic and personal gender equality. I use the term 'queer feminist' here also in its very broad sense according to which the perspective is taken that, firstly, gender and sexuality are central to any understanding of wider social and political processes, and secondly, these categories are to be studied as intersecting with other social inequalities like racialization, economic status, disabilities, etc. Since this is a rather recent development in the feminist debates (belonging to the 3rd wave of feminism), when referring to older literature (belonging to the 2nd wave) I will only use 'feminist'.

[2] See, e.g., [12, 11, 1, 2, 3].

[3] There is no need to worry about there not being a strict definition. Firstly, several proposals for specific definitions do exist in the quoted literature. But even if these may differ, this is just the exact same situation as for the term 'philosophical logic', which is sometimes understood as applying (non-classical) logics to philosophical problems, sometimes in a sense for which others use the term 'philosophy of logic'.

alisms", a special kind of dichotomies resulting from and simultaneously yielding the domination of one concept over the literal 'other'. This is said to be established especially by the conception of classical negation when $\sim p$ is interpreted as 'the other of p'. To clarify the points of criticism that are most significant for the present purpose, I will only mention three of the five features characteristic of dualisms that she claims to be inherent in classical logic and thus, to be responsible for a 'naturalization of domination', resulting from the omnipotence of classical logic.

Relational Definition (*Incorporation*): The other (e.g., 'women') is not defined in its own terms or positively but completely in dependence on the dominant side of the dualism as a lack or negativity (e.g., as 'not men').

Radical Exclusion (*Hyperseparation*): In a dualistic relationship the other is not only treated as different but as inferior, and to that end number and importance of differences between the sides is overemphasized by the dominant group and a possible overlap is denied. The dualistic pairs are constructed complementary, having "characteristics which exclude but logically require a corresponding and complementary set in the other" [9, p. 449]. The logical principle reflecting this is in Plumwood's opinion the principle of explosion: p and its other ($\sim p$) are to be kept at maximum distance; bringing them together yields the worst-case scenario of system collapse.

Homogenisation (*Stereotyping*): To confirm the 'nature' of the dualistic pairs both the dominant group as well as the dominated must appear maximally homogeneous. Therefore, stereotyping is used as an instrument of domination, whereby similarities are overemphasized, while differences within these groups are disregarded. Plumwood's interpretation of this feature of classical logic is much debated in the literature but in my opinion Ferguson's [4] extensive interpretation seems most reasonable in stating that the *Law of Excluded Middle*, representing the principle of exhaustivity of a domain, constitutes a likely candidate representing this dualistic feature in classical logic. There is no room for differentiation, everything other-than-p must fall under $\sim p$.

I think from this point of view so-called *contradictory logics* are interesting to consider for queer feminist theories for three reasons. Firstly, because they are not non-classical (in the usual sense of being a subsystem or an extension of classical logic) but *contra-classical*, secondly, because they fully accept contradictions instead of seeing them as "abnormalities", and thirdly, because on a bilateralist account of proof systems they can get rid of negation altogether. Therefore, while using the connexive logic `C` [13] here as an example to explain these points further and to motivate my account, I will deviate from its (and other contradictory logics') usual representation by considering a notion of contradiction that does *not* need to rely on negation as an underlying concept.

`C` is a contra-classical logic in that it validates theorems classical logic does not have. Thus, unlike most alternative logics considered by feminist logicians, in this case we have a logic which is not even a subsystem of classical logic. If we do consider Plumwood's criticism of classical logic valid, it seems desirable to free ourselves as rigidly as possible from it. Yet, there is a lot of evidence in the literature promoting 'usual' paraconsistent logics that reads almost apologetic toward giving up classical logic and that tries to argue their case by emphasizing that the differences to classical logic are as minimal as possible.[4] Arguments often go along lines where contradictions are seen sometimes literally as 'abnormalities' and classical logic as the logic for everything 'normal', while paraconsistent logics are retreats only for special cases. This is not the view taken in contradictory logics. Rather, they go beyond paraconsistency in that they are not only not explosive but actually have *contradictory theorems*, in the sense that there are formulas A for which there is both a proof of A and a proof of $\sim A$. With this feature they 1) certainly avoid the feature of Radical Exclusion and 2) seem to constitute a prime example of a desideratum implicit (and sometimes also explicit) in queer feminist theories: to *accommodate* contradictions instead of trying to avoid or overcome them.

It is this feature that not only makes contradictory logics interesting for queer feminist reasoning but also the other way around: it makes queer feminist theories interesting for contradictory logics because they constitute actual examples from philosophy of science, epistemology, etc. which explicitly endorse the existence of contradictions. Importantly, this happens on two levels: the theories are contradictory *and* the world itself is contradictory; the latter situation essentially being the cause for the former. Exemplary for promoting contradictory theories is, e.g., Harding [7, p. 180f.], who states that feminist epistemology "contains contradictions" and further that "its logic has surprising consequences: the subject/agent of feminist knowledge is multiple and contradictory, not unitary and 'coherent'". This has been criticized by some philosophers, e.g., Haack [6, p. 39] says about this quote that this would be "confusing" and "not very reassuring". However, this is not the case from the queer feminist perspective, rather it is almost natural that the theories must be contradictory because the situation in the world caused by an oppressive system is itself ultimately contradictory [7, 5]. This reasoning about social dimensions being contradictory has not received much attention in the area of formal logic despite there being a great interest in contradictions, paradoxes, etc. This lack of attention is in my opinion mainly due to the contingent fact that feminist philosophy (and social philosophy in general) has had a historically close connection to continental philosophy, i.e., to an area famously disregarding formal methods. I do not see any essential reason, though, why these two areas should exclude each other (nor do apparently the at least somewhat increasing number of researchers working on feminist logic).

Finally, if we consider a bilateralist interpretation of a contradictory logic like $\mathtt{C}$, it should be possible to get rid of (at least a primitive account of) negation completely, thereby avoiding the feature of Relational Definition. Specifically, this can be done by considering two derivability relations instead. Proof-theoretic bilateralism takes two dichotomic concepts, traditionally the speech acts of assertion and denial, strictly on a par and not one as reducible to the other. Here, instead of speech acts, I will rather consider the concepts of proof and refutation and show how these can be implemented proof-theoretically. Instead of conceiving contradictions in terms of negation, as it is most usually done,[5] we can then have contradictions by having A both provable and refutable in our system. Thus, at

[4] See [15] for an extensive discussion of this point.

[5] Also by Plumwood, see, e.g., comments like "contradiction being parasitic on negation" (p. 201) or "Contradiction is always characterized in terms of negation and the logical behaviour of contradictions is dependent on that of negation" (p. 204) in [10].

least prima facie it seems that such a bilateralist interpretation would be suitable to provide a dichotomic, yet not dualistic, representation of a logic.

As a suitable logic for this I will consider the negation-free fragment of the bi-connexive logic 2C as developed in [14] and show some features that are the outcome of dismissing negation.[6] I will show how the provable contradictions from 2C translate to contradictions without negation in the system considered here and also how we can still conceive of this system as connexive (given that many demarcations of connexive logic seem to depend on a negation). Since the strong negation in 2C serves as a toggle between proofs and refutations, one question that arises when getting rid of it is whether we lose the close connection between these concepts that is inherent in systems with this kind of 'toggle negation'. The answer is 'yes and no'. The concepts become more independent - which I do not consider unwanted from a bilateralist point of view - in that a proof (resp. a refutation) of A within the proof system cannot be immediately transferred into a refutation (resp. a proof) of A by using strong negation. Yet, as will be shown, by defining a notion of duality between formulas there is still a way to give a close relation between proofs and refutations in the system on a meta-level. I will also give a sketch of how we can use an annotation of the system with a two-sorted typed λ-term calculus to make these structural relations more visible and shed light on philosophical questions about proof identity. Another point that I will argue for is that in light of the connexive implications that 2C contains, it seems reasonable to implement certain relevance conditions for the implications.

Investigating whether and to what extent Plumwood's desiderata are met by this account, though, will show that we have good reasons to want to go further than that. As the "bi" in "bilateralism" clearly tells us, there is a *binarism* inherent in that picture. While Plumwood's remarks seem ambiguous on whether or not this aspect is to be retained or should be overcome, nowadays a strictly binary view with respect to concepts of gender but also others like race, sexuality, disability, etc. seems unsuitable. What seems rather appropriate here is to consider a wide spectrum accommodating fluidity for these concepts. In fact, the very harmful features that are ascribed to dualisms by Plumwood may be seen as inherent in a practice of 'mere' dichotomising differences.[7] Thus, as a tentative outlook I would like to consider whether a conception of *multilateralism*, as e.g. developed in [16], might provide a useful account for tackling this problem.

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[6]Note that in 2C, being constructive, the Law of Excluded Middle does not hold either, i.e., this is another favorable feature in terms of the criticism of Homogenization.

[7]On this point I would agree with a comment in a footnote in [2, p. 432] concerning the aspect of gender identity, namely that "it seems we are moving increasingly (though not quickly enough) towards rejecting Plumwood's dualisms, as she theorized them, altogether".

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A simulation of connexive logic based on pair sentential calculus

Tadao Ishii

Niigata University of Information and International Studies, Japan

ishii@nuis.ac.jp

1 Introduction

The sentential calculus with identity, **SCI** in short, is the most simplified version of R. Suszko's *non-Fregean logic* and can be obtained by adding the sentential identity connective $\equiv$ to classical logic [1]. **SCI** has the following identity axioms:

(E1) $A \equiv A$
(E2) $A \equiv B \rightarrow B \equiv A$
(E3) $A \equiv B \wedge B \equiv C \rightarrow A \equiv C$
(C1) $A \equiv B \rightarrow \neg A \equiv \neg B$
(C2) $A \equiv B \wedge C \equiv D \rightarrow (A\%C) \equiv (B\%D)$, where $\% \in \{\wedge, \vee, \rightarrow, \equiv\}$
(SI) $A \equiv B \rightarrow (A \rightarrow B)$

We will consider to deal with a simple Liar sentence : "This sentence is not true" in **SCI**. Let's define A="This sentence is true", then we have an equation of $A \equiv \neg A$ which means the referent of two sentences A and $\neg A$ are identical, but it is logically falsehood by (SI), i.e., $\neg(A \equiv \neg A)$ holds in **SCI**. To solve the matter, we have introduced a *referential relation* of pair-sentence $((\_)^i, (\_)^j)$ form , where i, j are some stage numbers, as the similar way to identity connective, i.e., $\neg(A \equiv \neg A) \iff (A^0, \neg A^1)$. More precisely speaking, we assume that for any formulas A appear in pair-sentence $(A^0, \neg A^1)$ form, if A^i holds in some situation with superscript i then its successor situation A^{i+1} is referred to $A^{i+1} := \neg A^i$. We have proposed a system **PSC** that just rejects the principle of identity "A is A". This treatment is similar to Gupta's sentence-definition with revision stage number [4], but the difference is our formalization was based on Suszko's **SCI** [5].

When doing logical reasoning, it is usually assumed that several fundamental postulates implicitly hold by a priori. These postulates are called Aristotle's classical three principles for thinking. The first *principle of identity* says that "A is always A and not being $\neg A$", the second *principle of contradiction* says that "A is not both A and $\neg A$", and the third *principle of excluded middle* says that "either A is B or A is $\neg B$". Then we get the following schemata from Aristotle's three postulates.

(AT1) $\neg(\neg A \rightarrow A)$
(AT1') $\neg(A \rightarrow \neg A)$
(AT1") $(A \rightarrow A)$
(AT2) $\neg((A \rightarrow B) \wedge (\neg A \rightarrow B))$
(AT2') $\neg(A \wedge \neg A)$

(AT3) $(A \to B) \vee (A \to \neg B)$
(AT3') $A \vee \neg A$

If we do not admit some of them, we will get several kinds of non-classical reasoning. But some postulates of (AT),(AT') and (AT2) are at all non-theorem of classical logic. Nowadays the standard notion of connexive logic can be characterized by the logical reasoning with external negation $\neg$ and connexive implication $\to$ (as non-symmetric) which satisfy Aristotle's non-classical postulates (AT1),(AT1') and also additionally the following similar Boethius' theses [7].

(BT) $(A \to B) \to \neg(A \to \neg B)$
(BT') $(A \to \neg B) \to \neg(A \to B)$

To simulate connexive reasoning in **PSC**, we have introduced the following interpretation of external negation for each connectives: for two stage numbers 0 and 1,

(1) $\neg(\neg A) \iff A$
(2) $\neg(A \wedge B) \iff \neg A \vee \neg B$
(3) $\neg(A \vee B) \iff \neg A \wedge \neg B$
(4) $\neg(A \to B) \iff A^0 \to B^1$
(5) $\neg(A, B) \iff (A^0, B^1)$

Then we get some extensions of **PSC** which admit the requirements in connexive logic and also can be seen not as one of four-valued logic, but as a classical two-valued logic according to Suszko's Thesis of bivalence [2].

2 PSC Logic

Let $\mathcal{L}_P =< \boldsymbol{FOR_P}, \neg, \wedge, \vee, \to, ((\_)^i,\ (\_)^j), \top, \bot >$ be a language of the sentential calculus with pair-sentence connective, where two constants are as usual $\top = p \vee \neg p$ and $\bot = p \wedge \neg p$ for some sentential variable p. We assume that every formula $A \in \boldsymbol{FOR_P}$ was assigned to a unique stage number 0 by a priori, i.e., $A^0 \in \boldsymbol{FOR_P}$. If we apply external negation to the above cases of (4) and (5), then under the insight into two formulas are incompatible each other [3], we interpret that the function of negation forces to split the stage number of formulas A, B in two $(0, 1)$ by keeping the internal logical operations. The pre-assigned stage number 0 may be omitted for the sake of simplicity. The **PSC** system is defined by the following axioms and inference rule:

(A1)-(A10) classical tautology axioms
(E1) (A, A) (E2) $(A, B) \to (B, A)$ (E3) $(A, B) \wedge (B, C) \to (A, C)$
(C1) $(A, B) \to (\neg A, \neg B)$
(C2) $(A, B) \wedge (C, D) \to (A\%C, B\%D)$ where $\% \in \{\wedge, \vee, \to, (\_, \_)\}$
(P1) $(A, B) \to (A \to B)$
(P2) $(A, B) \wedge (B \leftrightarrow C) \to (A, C)$
(Mp) $\dfrac{A \quad A \to B}{B}$

Here (E1) (A, A) means (A^0, A^0), but $\neg(A, A)$ means (A^0, A^1). Next we introduce some extensions of **PSC** as follows:

(P3) $(A,B) \wedge (A \leftrightarrow C) \to (C,B)$
(P4) $\neg(A \to B) \leftrightarrow (A^0 \to B^1)$
(P5) $\neg(A,B) \leftrightarrow (A^0,B^1)$
(S1) $\neg(\neg A, A)$
(S2) $\neg(A,\neg A)$
(S3) $(\neg(A \to B), A^0 \to B^1)$
(S4) $(\neg(A,B),(A^0,B^1))$
(1) $\mathbf{PSC_N} \stackrel{\text{def}}{=} \mathbf{PSC} \cup \{\text{(S3),(S4)}\}$
(2) $\mathbf{PSC_{CON}} \stackrel{\text{def}}{=} (\mathbf{PSC}/\{\text{(E2)}\}) \cup \{\text{(S1),(P3),(P4),(P5)}\}$

3 Connexive logic on PSC

There exist several kinds of representative theorems which may serve to characterize the connexive reasoning.
(cBT) $\neg(A \to \neg B) \to (A \to B)$ — converse of (BT)
(cBT') $\neg(A \to B) \to (A \to \neg B)$
(B3) $(A \to B) \to \neg(\neg A \to B)$ — variant of (BT)
(B4) $(\neg A \to B) \to \neg(A \to B)$ — variant of (BT')
(cB3) $\neg(\neg A \to B) \to (A \to B)$ — converse of (B3)
(cB4) $\neg(A \to B) \to (\neg A \to B)$
(AB1) $\neg((A \to B) \wedge (A \to \neg B))$ — Abelard's first principle

Proposition 1. The following are theorems of $\mathbf{PSC_N}$:

(1) $(A,B) \leftrightarrow (A \leftrightarrow B)$
(2) $(A^0 \to B^1) \leftrightarrow (\neg A^0 \to \neg B^1)$
(3) (P3),(P4),(P5),(S1),(S2)
(4) (AT1), (AT1')
(5) (BT), (cBT)
(6) (BT'), (cBT')
(7) (B3), (cB3)
(8) (B4), (cB4)
(9) (AT2), (AB1)

Proof. We show only the critical cases as follows: (2):1.$(A,B) \to (A \to B)$ (P1), 2.$\neg(A \to B) \to \neg(A,B)$ by 1-cont, 3.$(A^0 \to B^1) \to (A^0,B^1)$ (P4),(P5), 4.$(A^0, B^1) \to (\neg A^0, \neg B^1)$ (C1), 5.$(\neg A^0, \neg B^1) \to (\neg A^0 \to \neg B^1)$ (P1), 6.$(A^0 \to B^1) \to (\neg A^0 \to \neg B^1)$ by 3–5,$\to$trans, 7.$(\neg A^0 \to \neg B^1) \to (A^0 \to B^0)$ by similar to 1–6. (3) (S1):1.$(\neg A, A) \to (\neg A \leftrightarrow A)$ by (1), 2.$(\neg A \leftrightarrow A) \leftrightarrow \bot$, 3.$(\neg A, A) \to \bot$ by 1,2,$\to$trans, 4.$\neg\bot \to \neg(\neg A, A)$ by 3-cont, (S2):1.$\neg(\neg A, A)$ (S1), 2.$\neg(\neg A, A) \leftrightarrow (\neg A^0, A^1)$ (P4), 3.$(\neg A^0, A^1) \to (\neg\neg A^0, \neg A^1)$ (C1), 4. $\neg\neg A^0 \leftrightarrow A^0$ (cl-taut), So $(A^0, \neg A^1) \leftrightarrow \neg(A, \neg A)$ by 1–4,(P3). (4) (AT1):1.$\neg(\neg A, A) \leftrightarrow (\neg A^0, A^1)$ (S1),(P5), 2.$(\neg A^0, A^1) \leftrightarrow \neg(\neg A \to A)$ (P4),(P5) so we get the result. (5) 1.(A,A) (E1), 2.$\neg(B,\neg B) \leftrightarrow (B^0, \neg B^1)$ (S2),(P5), 3.$(A,A) \wedge (B^0, \neg B^1) \to (A^0 \to B^0, A^0 \to \neg B^1)$ (C2), 4.$(A^0 \to B^0, A^0 \to \neg B^1) \to (A \to B, \neg(A \to \neg B))$ (P4),(P2),5.$(A \to B, \neg(A \to \neg B))$ by 1–4, Mp, 6.$(A \to B) \leftrightarrow \neg(A \to \neg B)$ by 5,(1). (6) 1.$(A^0 \to B^0, A^0 \to \neg B^1)$ (5), 2.$(A^0 \to B^0, A^0 \to \neg B^1) \to (\neg(A^0 \to B^0), \neg(A^0 \to \neg B^1))$ (C1), 3.$\neg(A^0 \to \neg B^1) \leftrightarrow \neg(\neg(A^0 \to \neg B^0)) \leftrightarrow (A^0 \to \neg B^0)$ (P5),(cl-taut), 4.$(A^0 \to \neg B^0, A^0 \to B^1)$ 1–3,(E2), 5.$(A \to \neg B) \leftrightarrow \neg(A \to B)$ by 4,(1). (7) 1.$(A^0 \to B^0, A^0 \to \neg B^1)$ (5), 2.$(A^0 \to \neg B^1) \leftrightarrow (\neg A^0 \to \neg\neg B^1) \leftrightarrow (\neg A^0 \to B^1)$ (2),(cl-taut), 3.$(A^0 \to B^0, \neg A^0 \to B^1)$ by 1,2,(P2), 4.$(A \to B) \leftrightarrow \neg(\neg A \to B)$ (P4),(P5). (8) 1.$(A \to B, \neg A^0 \to B^1)$ (7), 2.$(\neg A^0 \to B^1, A \to B)$ by 1,(E2), 3.$(\neg(\neg A^0 \to B^1), \neg(A \to B))$ by 2,(C2), 4.$\neg(\neg A^0 \to B^1) \leftrightarrow \neg(\neg(\neg A^0 \to B^0)) \leftrightarrow (\neg A \to B)$ (P4), (cl-taut), 5.$(\neg A \to B, \neg(A \to B))$ by 3,4,(P3), 6.$(\neg A \to B) \leftrightarrow \neg(A \to B)$ (1). (9) (AT2):1.$\neg((A \to B) \wedge (\neg A \to B)) \leftrightarrow \neg(A \to B) \vee \neg(\neg A \to B)$ (cl-taut), 2.$\neg(\neg A \to B) \leftrightarrow (\neg A^0 \to B^1) \leftrightarrow (A \to B)$ (7), 3.$\neg(A \to B) \vee (A \to B) \leftrightarrow \top$ by 1,2,(cl-taut). □

Proposition 2. The following are theorems of $\mathbf{PSC_{CON}}$:

(1) (AT1), (AT1')
(2) (BT), (BT')
(3) (B3), (B4)

Proof. (1) (AT1):1.$\neg(\neg A, A) \leftrightarrow (\neg A^0, A^1)$ (S1),(P5), 2.$(\neg A^0, A^1) \to (\neg A^0 \to A^1)$ (P1), 3. $\neg(\neg A \to A)$ by 1,2,(Mp),(P4), (AT1'):1.$\neg(A, \neg A)$ by Prop 3.1(3), so we get by similar to (AT1). (2) (BT):$(A \to B, \neg(A \to \neg B))$ by Prop 3.1(5), we get $(A \to B) \to \neg(A \to \neg B)$ by (P1). But the converse does not hold because we eliminate (E2) axiom from $\mathbf{PSC_N}$. The other cases of (BT') and (3) are similar to (BT). □

4 Semantics

We will check the validity of (AT1) and (BT) by using a classical truth table as follows: [6]

Table 1: (AT1): $\neg(\neg A \to A) \iff (\neg A^0 \to A^1)$

A^0	$\neg A^0$	$\neg(\neg A^0 \to A^0)$	$A^1 \stackrel{\text{def}}{=} \neg(A^0)$	$\neg A^0 \to A^1$
1	0	0	0	1
0	1	1	1	1
		classical		$\mathbf{PSC_{CON}}$

Table 2: (BT): $(A \to B) \to \neg(A \to \neg B) \iff (A^0 \to B^0) \to (A^0 \to \neg B^1)$

A^0	B^0	① $A^0 \to B^0$	② $\neg(A^0 \to \neg B^0)$	③ $A^0 \to \neg B^1$	① → ②	① → ③
1	1	1	1	1	1	1
1	0	0	0	0	1	1
0	1	1	0	1	0	1
0	0	1	0	1	0	1
					classical	$\mathbf{PSC_{CON}}$

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What medieval logicians have to say about the basic principles of connexive logic

Wolfgang Lenzen

University of Osnabrück, Germany

lenzen@uos.de

1 The basic principles of connexive logic

Connexive logic is closely linked to the history of logic, in particular to the ancient logicians *Aristotle* and *Chrysippus*, and the early medieval *Boethius*. According to Storrs McCall, connexive implication was first defined by a Stoic logician whose basic idea had been described as follows:

> And those who introduce the notion of connection say that a conditional is sound when the contradictory of its consequent is incompatible with its antecedent.

While the concurring conceptions of *material* and *strict* implication were attributed to *Philo* and *Diodorus*, respectively, Sextus Empiricus didn't mention the name of the logician who defended the third conception of *connexive* implication. According to the Kneales, however, this person most likely was *Chrysippus*. The main difference between Diodorean and Chrysippean implication can be illustrated by the example

Diod   If atomic elements do not exist, then atomic elements do exist.

Diodorus accepted Diod as "sound" because the existence of physical atoms is (at least in the Stoics' opinion) *necessary*. Hence, the antecedent of Diod is *impossible* while its consequent is necessary, so that it can never happen that the antecedent be true and yet the consequent false. For Chrysippus, however, Diod fails to be "sound" because the contradictory of its consequent is *identical* with its antecedent and hence there is no real conflict, no "incompatibility" between these two propositions.

As has been argued in [7], what makes Chrysippean implication *connexive* is not the basic idea of the above quoted definition which might be formalized, with 'I' abbreviating the relation of incompatibility, as follows:

Chrys 1 $(p \to q) \Leftrightarrow \mathrm{I}(p, \neg q)$.

The real source of the connexivity of Chrysippus's conception rather lies in the assumption that the relation 'I' is *strictly anti-reflexive*. That is, for Chrysippus *no proposition whatsoever* is *incompatible* with itself:

Chrys 2 $\neg\mathrm{I}(p, p)$.

Substituting '$\neg q$' for 'p' in Chrys 2 yields $\neg\mathrm{I}(\neg q, \neg q)$, which, according to Chrys 1, means that $(\neg q \to q)$ is always false. Following McCall, the latter principle shall be referred to as *Aristotle's first thesis*:

Arist 1 $\neg(\neg q \to q)$.

As a matter of fact, Aristotle used this principle in order to prove another characteristic law of connexive law, saying "that two implications of the form 'If p then q' and 'If not-p, then q' cannot both be true"([11], p.415). In accordance with McCall's terminology, this principle shall be referred to as *Aristotle's second thesis*:

ARIST 2 $\neg((p \to q) \wedge (\neg p \to q))$.

ARIST 1 is often formulated "passively" by saying that a proposition cannot *be implied* by its own negation. Similarly, ARIST 2 might be paraphrased as saying that no proposition q can *be implied* (or entailed) by both of two contradictory propositions p and $\neg p$. Let us, however, also consider corresponding "actively" formulated variants saying that no proposition *implies* its own negation, and that no proposition p *implies* both of two contradictory propositions q and $\neg q$:

ABEL 1 $\neg(q \to \neg q)$

ABEL 2 $\neg((p \to q) \wedge (p \to \neg q))$.

While ABEL 2 is usually referred to as '*Boethius'* Thesis', it shall here be called *Abelard's* (second) thesis because the "Palatine master" explicitly defended these principles (together with their Aristotelian counterparts) in his *Dialectica*,[2]. The aim of this paper is to examine the views of medieval logicians not only concerning the connexive principles ARIST 1,2, ABEL 1,2, CHRIS 1,2, but also concerning "anti-connexive" principles like "Ex impossibili quodlibet", "Necessarium ad quodlibet", and "Ex contradictione quodlibet":

2 Logicians from the 12th and 13th century

In [8] and [9] it has been argued that Peter Abelard (1079-1142) was the first logician who tried to defend Aristotle's theses against counter-examples as they had been discovered by contemporary logicians. Abelard clearly recognized that the "usual" conception of implication in the sense of

STRICT $(p \to q) \Leftrightarrow_{\text{df}} \neg\Diamond(p \wedge \neg q)$

would give rise to the validity of "Ex impossibili quodlibet":

EIQ If $\neg\Diamond p$, then $(p \to q)$, for any q

E.g., 'Socrates is a stone' would entail 'Socrates is an ass' for "it is impossible that Socrates should be a stone, and so impossible that he should be a stone without being an ass" [6]. More generally, if p is impossible, then, for any q, it is "impossible that the antecedent should be true without the consequent", or, as Abelard put it: "Quod enim omnino non potest esse, et sine ille non potest esse"([2], p.285).

Abelard therefore suggested to replace STRICT with the more demanding condition that "not only the antecedent cannot be true without the consequent, but also the [truth of the] antecedent requires the [truth of the] consequent *by itself*". As a typical example of this stronger conception of a "natural" implication he mentions 'If Socrates is a man, Socrates is an animal'. A corresponding implication with a *negative* consequent like 'If Socrates is a man, Socrates is not a stone' is *not* considered by Abelard as "naturally" valid, because: "Not being a stone does not follow in the appropriate way from being a man, even though it is inseparable from being a man. It does not follow in the appropriate way since it is no part of the nature of a man that he not be a stone"([10], p.392).

However, Alberic of Paris developed an "embarrassing" argument which refuted the connexive principle ABEL 1. Since Alberic's proof made use only of logical principles which Abelard regarded as indispensable, namely, the laws of *conjunction*, *contraposition*, and *transitivity* of implication, "[...] confronted with this argument Master Peter essentially threw up his hands and granted its necessity"([10], p.395).

Somewhen in the 12$^{\text{th}}$ century, clever logicians discovered a *proof* of the principle "Ex *contradictione* quodlibet" saying that any consequent q follows from a self-contradictory antecedent like $(p \wedge \neg p)$:

ECQ $(p \wedge \neg p) \rightarrow q$, for any q.

A familiar version of this proof may be found in Alexander Neckham's *De Naturis rerum* composed around 1180. The proof is based on the laws of conjunction plus so-called *disjunctive syllogism* saying that if a disjunction is true and if one of its disjuncts is false, then the other disjunct has to be true.

In his detailed commentary on Aristotle's logic, Robert Kilwardby (1222-1277) attempted to save ARIST 1,2, ABEL 1,2 from refutation. On the one hand, Kilwardby was well aware of the fact that the connexive principles stand in conflict with "Ex impossibili quodlibet" and the counterpart "Necessarium ad quodlibet":

NAQ If $\Box q$, then $(p \rightarrow q)$, for any p.

According to Kilwardby, however, inferences based on EIQ and NAQ are not "naturally valid" but only "accidentally valid"; therefore, in his opinion, they do not genuinely affect Aristotle's theses. On the other hand, Kilwardby clearly saw that the standard laws of disjunction entail $p \Rightarrow (p \vee \neg p)$ and $\neg p \Rightarrow (p \vee \neg p)$, which constitutes a counter-example to ARIST 2. Thus, Kilwardby eventually admitted: "So it should be granted that from the impossible its opposite follows, and that the necessary follows from its opposite"([4], p.86). As a special instance of ECQ, one obtains the following Kilwardbyan principle:

KILW $(p \wedge \neg p) \rightarrow \neg(p \wedge \neg p)$.

3 Logicians from the 14th and 15th century

In *On the Purity of the Art of Logic*, Walter Burley (ca. 1275–1345) considered several propositions which entail their own negation: "For example, it follows: 'You know you are a stone; therefore, you do not know you are a stone', because the antecedent includes opposites."([1], p.156–7).

More generally, Burley recognized that each proposition, which "includes" or entails two opposites q and $\neg q$, entails its own negation:

> From these rules [of contraposition and transitivity], the claim is proved as follows: If some proposition includes opposites, it implies either of them. Since therefore, from the opposite of a consequent there follows the opposite of its antecedent, from the opposite of either of those contradictory consequents there must follow the contradictory of the antecedent. Since therefore, the opposite of either one follows from the same antecedent, and whatever follows from the consequent follows from the antecedent, from that antecedent there must follow its contradictory.([1], p.157)

But for Burley, some propositions *do* include opposites: "All people generally agree on this"([1], p.159). In particular, as a corollary of the general laws of conjunction, the self-contradictory $(q \wedge \neg q)$ entails both q and $\neg q$, so that ARIST 2 has to be restricted as follows:

> Suppose someone says contradictories do not follow from the same antecedent. For in that case the same thing would follow from contradictories which seems to be contrary to the Philosopher in Prior Analytics who says the same consequent does not follow from the same antecedent affirmed and denied. I say that the same consequent does not follow from the same antecedent affirmed and denied, unless the opposite of that consequent includes contradictories. And this is how Aristotle's statement has to be understood.([1], p.160)

Hence, Aristotle's second thesis does *not hold* if the opposite of the consequent q is impossible, i.e., if q itself is *necessary*. In a similar way, John Buridan (ca. 1300–1358) pointed out that the connexive principle Abel 1, according to which no proposition entails its own negation, holds only for *self-consistent* antecedents. He remarked rather incidentally that a "*possible* proposition never entails its own contradictory", where the editor of the English translation of the *Sophismata*, G. E. Hughes, hastened to add:

> Note that the principle appealed to is not that no proposition whatsoever can entail its own contradictory, but only that no possible proposition can do so. This is a standard principle of modal logic; an alternative formulation is, 'Any proposition that entails its own contradictory is impossible'.([4], p.86)

A much more extensive discussion of the connexive principles may be found in the works of the so-called *Pseudo-Scot*. "Quaestio III" of his commentaries on Book 2 of Aristotle's *Prior Analytics* is devoted to the question "Whether the same can follow from both of two contradictory propositions" (cf. [3], p.183). On the one hand, according to Naq, a *necessary* proposition like 'God exists' follows from any other proposition, hence both from 'God is a substance' and from 'God is not a substance'. On the other hand, the tautological disjunction 'Socrates is running, or Socrates is not running' follows from each of its disjuncts. Apart from these exceptions, however, Aristotle's thesis "Ad idem esse, et non esse, non sequitur idem" holds; in particular it holds for *categorical* propositions like '*B est magnum'* and '*B non est magnum'*.

In *Logica Parva*, Paul of Venice (ca. 1370–1429) considered the following objection which other logicians had raised against Eiq and Naq:

> It does not follow, 'Some man is a donkey; therefore, no man is a donkey'. And this is argued according to both rules; therefore, both rules are false. The inference holds with regard to the minor premise; and the major premise I prove. First, because from one of two opposites the remaining one does not follow. Second, because the contradictory of the consequent stands with [i.e., is compatible with] the antecedent insofar as it is interchanged [i.e., equivalent] with that very proposition.([12], p.185)

The second argument is very interesting because it shows that some of Paul of Venice's contemporaries believed (just like Chrysippus) that *each* proposition is *compatible* with itself. Paul, however, replied that:

> [. . .] it is not absurd from one of two opposites to infer the other one – given that the proposition in question is impossible. I deny, however, that the opposite of the consequent stands with the antecedent. And I deny [the inference: the opposite of] 'the consequent is interchanged with the antecedent; therefore, it stands with that same proposition'. Whence I say that any proposition in the world follows from that proposition ['A man is a donkey']; and any proposition is repugnant to that that same proposition. Indeed it is repugnant to its very self because it implies the opposite of its very self.([12], p.285–6; my emphasis)

Hence, according to Paul of Venice, principle CHRYS 2 has to be restricted to self-consistent propositions. In *Logica Magna*, Paul made another interesting observation when he discussed the issue whether a conditional can ever make an "positive assertion". Some medieval logicians believed that a (strict) implication *never* entails the truth, or falsity, of the antecedent or the consequent. However, Paul recognized that each conditional $p \to q$ can be equivalently replaced by the assertion that the *disjunction* $(\neg p \vee q)$ is *necessary.* From this it follows that the instance $p \to \neg p$ is equivalent to $\Box(\neg p \vee \neg p)$, and hence, in view of the idempotence of the disjunction operator, equivalent to $\Box\neg p$. Thus, instead of ABEL 1, only the following principle holds:

PAUL $(p \to \neg p) \Leftrightarrow \neg\Diamond p$.

From this Paul further concluded that "there are some conditionals expressed by 'if' which formally entail a contradiction", e.g., "If you are not other than yourself, you are other than yourself"([5], p.41). More generally, any *conditional* of type $(p \to \neg p)$ where p is a *tautology* and where $\neg p$ is hence *self-contradictory*, *entails* a contradiction, namely $\neg p$, and is therefore itself self-contradictory!

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Formalization of the Chysippus conditional

Piotr Łukowski

Jagiellonian University, Poland,

piotr.lukowski@uj.edu.pl

Sextus Empiricus in his books Against the Logicians and Outlines of Pyrronism, represents the difference of opinion between Diodorus Cronus and Filo, on the conditions for the truth of a conditionals, which we use in our everyday thinking. Philo understands conditional exactly as classical implication is understood today. Diodorus criticizes this by giving a number of examples refuting the assumption that everything follows from falsity. Also, the assumption of the truth of an implication when both its elements are true is unacceptable to Diodorus. His counter-examples are quite convincing. Diodorus agrees with Philo only in that an implication is false when its antecedent is true and its consequent is false. His conditions for the truthfulness of the implication are more complex and closer to our thinking, although not entirely consistent with it. He considers an implication to be true when, by necessity, if the antecedent is true, then the consequent is also true. Unfortunately, Diodorus understood necessity temporally: now and always in the future when the antecedent is true, the consequent is true. This has its undesirable consequences. Therefore, a better understanding of necessity is that of Chrysippus: the truthfulness of the predecessor cannot occur simultaneously with the falsity of the successor. This Chrysippian understanding is believed to apply to contradiction. The choice of implication of the Philo type underpinned the remarkable development of mathematics and the unique role of classical logic as a meta-logic for developing non-classical logics. Although it seems that the implication cannot be extensional. Otherwise it is a simple disjunction. The actual nature of human implication is intensional.

The presentation will propose such a formalization of implication that seems to satisfy the suggestions of Diodorus (without temporal understanding of necessity) and Chrysippus. Moreover, it also conforms to Aristotle's postulates, which are nowadays considered the basis of connexive logics – the actual human implication should be connexive. It is likely that the insightful Aristotle, who lived many years before Philo, would not have recognized Philo's conditions of the truth of implication as a correct definition of implication. His "connexive" postulates for implication preclude its material, i.e. extensional, flat understanding.

We will attempt to reconstruct the logic of content to a form that is as close as possible to both our thinking and classical logic. Among the classical connectives, some seem close to our thinking, others foreign to it. The former include connectives of negation, conjunction and disjunction. The latter, the connectives of implication and equivalence. Therefore, the connectives of classical implication and equivalence are replaced by the intensional T-implication and T-equivalence. The semantics is Fregean i.e. with the content implication and the synonymy. Thus every model interprets negation, conjunction, disjunction, T-implication, T-equivalence, content implication and synonymy (i.e. a conjunction of two mutually inverse content implications). Our semantics consists of a class of (Fregean) models and one (Fregean) mapping. This one mapping assigns a content to each sentence and that is why it is called sentence understanding. Since it is one, each sentence has its own specific, unchanging content in our semantics. Naturally, sentences having the same logical value of true or false, i.e. T-equivalent, may have different contents, i.e. they may not be synonymous. Negation, conjunction and disjunction are interpreted in a classical way. These connectives are used to define T-implication and T-equivalence. However, none of these two just defined connectives is extensional because in their definitions some specific subclass of the class of all models is used. Thus, if T-implication is satisfied in one model, then it is satisfied in every model of this selected subclass.

The classical characterization of connectives of negation, conjunction and disjunction means that all of them are extensional. Moreover, every sentence in the shape of a classical tautology, i.e., every logical truth, that can only contain these three connectives, is accepted in every model. Thus, if our reasoning is based on premises containing only these three connectives, it should be limited to one model. For example, if I say that I will be in room 30 or lecture hall 27, and later I happen to not be in room 30, then one can infer that I am in lecture hall 27. This reasoning uses one particular understanding of sentences and one particular state of affairs. The well-known problems with the classical connectives of implication and equivalence are that, being extensional, they are supposed to represent the non-extensional implication and equivalence of our thinking. Therefore, unlike sentences containing a classical connectives of implication or equivalence, sentences with only negation, conjunction and disjunction do not produce paradoxical consequences. The well-known, definition of classical implication by using disjunction and negation extensionalizes implication: using only extensional connectives defines an extensional connective, although the defined connective should be intensional. Such a flattening understanding of the implication makes quite reasonable Aristotle's postulates incomprehensible. Rejecting (A implies not-A) and (not-A implies A), i.e. not-A and A, respectively, is clear nonsense due to the arbitrariness of A. Similarly, accepting not-(A implies not-A) and not-(not-A implies A), i.e. A and not-A, respectively, is clear nonsense also. Thus, "according to Aristotle" we should simultaneously reject and accept each sentence A.

In our everyday thinking, we use implication, which is not extensional, but intensional: we will say that A implies B when it is unthinkable for A to occur and B not to occur; when some content (maybe state of affairs) of A necessarily entails another content of B. For example, we will say without a doubt that, if Tweety is an ostrich, then Tweety is a bird, because it is inconceivable for Tweety to be an ostrich without being a bird. In other words, there is no such a model for an ostrich not to be a bird. It is, on the other hand, conceivable that Tweety being a bird is not an ostrich. That is because there is such a model that some bird is not an ostrich. Therefore, we will not say that if Tweety is a bird, then Tweety is an ostrich. Moreover, with a given understanding of sentences, if an implication is true (or false), then it is so in any model that preserves that understanding. This means that acceptance of the implication "if Tweety is an ostrich, then Tweety is a bird" does not depend on whether Tweety is an ostrich or not. Following this idea, we defined above the new T-implication.

Choosing and establishing some one particular mapping as the understanding of sentences means that in all models every sentence A has the same meaning/-content. Moreover, the choice of function is such that the meanings of sentences are consistent with how we understand sentences. However, the class of all models includes some in which the meaning attributed to sentences is neither consequent nor consistent with our understanding of them. For example, there is probably a model in which the sentences “Tweety is an ostrich” and “Tweety is a chair” will be considered true. For this reason, it is necessary to make a selection of models that will result in only those whose sentence meanings are consistent and coherent with each other. For example, if someone is shaggy, he cannot be bald. All these models respecting the accepted meanings of sentences form the selected subclass. With this (first) selection, T-implications defined using the subclass express the true analytical and structural conditionals mentioned in the beginning. Naturally, models belonging to the selected subclass can undergo further (second, third,...) selection so that T-implications also express the true empirical or thetic conditionals. Both the choice of one mapping and the one specific subclass serve to avoid combinatorial randomness and arbitrariness in assigning meanings to sentences. This is the way to guarantee that the resulting logic can be effectively named “content”.

T-implication as well as T-equivalence is not extensional. They are also not hyperintensional, but intensional. The information about the logical value of the antecedent and consequent does not determine whether the entire T-implication is true, however, with three exceptions: 1. when the antecedent is logically false; 2. the consequent is logically true; 3. the antecedent is true and the consequent if false. Such T-implication is not vacuously satisfied: if a T-implication with a false antecedent is true in a model, it is not due to the falsity of the antecedent, but because this T-implication expresses a conditional common to a selected class of models defined by the fixed mapping of sentence understanding. Thus, the "T" in the name does not mean “truth-functional”, but that T-implication says what a given content really (truly) implies. Thus, in the definition of T-implication, the expression “truly implies” cannot therefore be replaced by “truth-functionally implies”. The choice of the letter "T" is also a gesture referring to the Angell’s operator T extending his AC system. Similarly, for two sentences to be T-equivalent, their simultaneous truth (but not logical truth) or falsehood (but not logical falsehood) is not enough. It is necessary that there is no model accepting one sentence and rejecting the other. Unlike the pair of sentences A and (A or B), sentences not-(A or B) and (not-A and not-B) are T-equivalent. In the first pair of sentences, only A truly implies (A or B), of course, if e.g. B in not not-A. The sentences “Today is Christmas Eve” and “Counting from today, the ninth day is New Year” can be an example of natural language sentences which are T-equivalent.

It is noteworthy that T-implication fulfills those postulates made in antiquity by Diodorus, Chrysippus and Diogenes. First, the truthfulness of a T-implication does not depend solely on the logical value of the antecedent and consequent – it is not possible for the antecedent to be true while the consequent is false. Second, if the T-implication is true in some model of the selected subclass, it is true in any model of that subclass. This is in line with our daily thinking. After all, we will consider as true the implication “If Tweety is an ostrich, then it is a bird” even if Tweety is a chair. Third, contradiction does not truly imply anything. This property follows from the first condition of the definition of T-implication, which assumes that there is a model in the subclass in which the antecedent of

T-implication is true. And this means, the non-contradiction of the antecedent. Moreover, it is not difficult to notice that such understanding of T-implication directly relates to Aristotle's and Boethius theses defining connexive logic. It can even be said that this very idea of Aristotle is embodied literally in this definition.

Until now, the connective of disjunction has been consistently understood as extensional. This is not surprising, since as extensional it is intuitive for us. For example, the sentence (A or B) = "For vacation we will go to the sea or the mountains" only expresses that the vacation will be spent on the seaside or in the mountains, or (partly) on the seaside and (partly) in the mountains. Naturally, we can understand this example disjunction as the classical, and so extensional, implication (not-A implies B), which is classically equivalent to (not-B implies A). Thus, the sentence (A or B) means nothing more, than "If we don't go to the sea, we'll go to the mountains" and "If we don't go to the mountains, we'll go to the sea." This is so because the content of a sentence (A or B) only expresses the relations between the logical values of sentences A and B. Such extensionally understood disjunction is commutative.

These considerations are in line with the approach of Ajdukiewicz proving that our human conditional is the same as the classical material implication (Ajdukiewicz 1956). In Ajdukiewicz's argumentation, the disjunction of consequent and negation of the antecedent of an implication played a central role. Following the same line, the disjunction corresponding to T-implication can be reconstructed. Such a disjunction will necessarily be intensional.

Thus, there is possible another, intensional understanding of disjunction, in which its content expresses something more than the relations between the logical values of disjuncts. For example, let C = "We will go on vacation" and D = "We will go to the seaside". Then, the sentence (not-C T-or D) = "We won't go on vacation or go to the seaside" says that (C T-implies D) = "If we go on vacation it's only to the sea," and the same, (not-D T-implies not-C) = "If we don't go to the sea, we won't go on vacation at all." This is because content of D is, in a sense, part of content of C. Otherwise, the sentence (not-D T-or C) = "We won't go to the sea or go on vacation" says that (D T-implies C) = "If we go to the sea, we will go on vacation (i.e. not to work)" = "If we go to the sea, it's only to rest, and not for working", and also (not-C T-implies not-D) = "If we don't go on vacation, we won't go to the sea." This is because content of C is, in a sense, part of content of D. In both cases we are dealing with some other disjunction than "or". We used the symbol "T-or" to denote it, because it is indeed definable with T-implication. Neither in (not-C T-or D), nor in (not-D T-or C) are C and D interchangeable. In this sense intensional T-disjunction is not commutative.

Of course, due to the negation of only one of its disjuncts, this new disjunction is not commutative. Instead, it is intensional – the truth of a T-disjunction does not depend solely on the logical values of its disjuncts. Moreover, directly from conditions characterizing T-implication and T-disjunction, respectively, we have in the example that: (not-C T-or D) = (C T-implies D) and (not-D T-or C) = (D T-implies C). The relevant laws are proven below.

On the selected class of models depends what kind of knowledge is represented by conditionals: physical, historical, legal, popular, etc. However, there are also conditionals that express knowledge about logical truths and rules. These ones do not require any selection of models, since logical truths and rules are expressed in every model. Therefore, in order to recognize only these truths and rules, the subclass should be extended to the class of all models.

Thus two inferences, truth and strictly-truth, are defined. The vast majority of their properties seem to be in line with our thinking.

Truth-inference:

from falsehood nothing
from contradiction nothing
tautologies from empty set
rule of addition
rule of reflexivity with limits
intensional
monotonic
non-paraconsistent
CONNEXIVE

Strictly-truth-inference:

from empty set nothing
from falsehood nothing
from contradiction nothing
tautologies from no set
rule of addition
rule of reflexivity with limits
intensional
monotonic
non-paraconsistent
CONNEXIVE

There are also defined two quasi-classical valuations with appropriate inferences. They are connexive and, moreover, have properties typical of our human thinking.

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On the history and motivations of connexive logic (extended abstract)

Hans Rott

University of Regensburg, Germany

hans.rott@ur.de

This extended abstract reads more like a plan for a research project. It lists a fairly large number of claims many of which will be controversial and need to be substantiated.

1. Connexive logic is special in that it has *historical motivations*.
   - 1.1. Much of it centers around, and its discussion is dominated by, a small number of 'theses' that are attributed to some historically eminent philosopher-logicians (Aristotle, Boethius, Abelard).
   - 1.2. The consensus now is that the core of connexive logic is defined by Aristotle's First Thesis and Boethius's Theses.
   - 1.3. A minority of researchers also considers at least one of Aristotle's Second Thesis, Abelard's Thesis and the Converse Boethius Thesis as (core?) connexive thesis.
   - 1.4. The transcription of the theses into modern logical notation is neither obvious nor precisely determined.[8]
   - 1.5. In the original authors, there is comparatively little text concerning the theses, and they are often only used rather than declared logical principles.
   - 1.6. Still one can, and perhaps should, try to make explicit the motivations of the great old authors.
   - 1.7. However, they had no clear distinction between metalanguage (with predicates like 'implies', 'entails' etc.) and object language (with the conditional connective '$\rightarrow$'), and they had no fully developed systems.
   - 1.8. Their works serve as an excellent inspiration, but have only limited normative force for contemporary connexive logicians.
   - 1.9. The early history of connexive logic is a most interesting field of research which, of course, has a decidedly historical perspective that must avoid superimposing a modern understanding of logic onto Aristotle, Boethius and Abelard.
2. *Modern connexive logic* started in the middle of the 20th century, sometime between 1930 and 1962 (after connexive logic had been down for some 800 years).
   - 2.1. The founding authors were E. J. Nelson (1900–1988), R. B. Angell (1918–2010) and S. McCall (1930–2021).
   - 2.2. Though some prominent authors of modern connexive logic have very pronounced historical interests (e.g., McCall and Sylvan), their ambitions are not primarily historical, but they want to develop 'good' connexive systems.

[8] Wansing [8] mentions at least five versions of Boethius's Thesis. An important proposal how to standardise the terminology regarding connexive logic is made by Wansing and Omori [9].

2.3. Modern connexive logic can be viewed as an enterprise of *logic revision*: revise or "contract" classical logic (or some other reference logic) in such a way that the theses mentioned above can be included, without making the logical theory trivial. (Local inconsistencies may be allowed.)
2.4. Logic revision can be performed as a purely mathematical exercise. It is interesting, challenging, sometimes beautiful, but in itself it does not carry any meaning.
2.5. Still one should make explicit the philosophical motivations of Nelson, Angell, McCall and their successors.

3. Various formal representations of *incompatibility* can be distinguished (for the purposes of connexive logic). They are significant for Boethius's Thesis, Aristotle's Second Thesis and Abelard's Thesis. (Aristotle's First Thesis only needs a non-relational notion of impossibility.)
   3.1. Perhaps the most intuitive representation of incompatibility is that of simultaneous unsatisfiability, for example, of $A \to C$ and $A \to \neg C$. This is Kapsner's 'Unsat2' rendering of Boethius Thesis.
   3.2. The same (or a similar) idea can be put into the object language directly by using conjunctions and requiring that $\neg((A \to C) \wedge (A \to \neg C))$ be an axiom scheme or a theorem of the formal system. This is called 'Abelard's First Principle' (Martin).
   3.3. A very similar symbolization uses the material conditional: $((A \to C) \supset \neg(A \to \neg C))$. This is called 'Weak Boethius Thesis' (Pizzi).
   3.4. The same (or a similar) idea can also be put into the object language requiring that $((A \to C) \to \neg(A \to \neg C))$ be an axiom scheme or a theorem of the formal system. This is commonly called 'Boethius Thesis'.
   3.5. Boethius Thesis is nice for the logician because it uses only two connectives rather than three. The other versions are nice because we have firmer intuitions about first-degree (non-nested) conditionals than about higher-degree (nested) conditionals. It is not clear which advantage is greater.
   3.6. The Unsat notion of incompatibility is a semantic one, in contrast to the other three syntactic notions that concern properties of formal systems (which need not be interpreted).
   3.7. Similar variations can be considered with respect to Aristotle's Second Thesis, which concerns negations of antecedents rather than negations of consequents (see 11. below).

4. Connexive logic can be considered as a *theory of conditionals*.
   4.1. Conditionals, i.e., 'if ... then' sentences or their symbolization by '$\to$' in a regimented object language, play a central role in the theses mentioned above.
   4.2. The conditional of connexive logic contrasts with the material conditional and the strict conditional in that the antecedent is supposed to be *connected with* or *relevant to* the consequent.
   4.3. Like relevant logic, connexive logic can be called a 'sociative logic' (Sylvan 1989). Their motivations are similar.
   4.4. The conditional (of connexive logic) expresses a binary relation between antecedent and consequent that cannot be reduced to a monadic property

of either the antecedent or the consequent. It is essentially relational.

5. At least two kinds of conditionals need to be distinguished (for the purposes of connexive logic). Let us call them *entailment-representing conditionals*[9] and *ordinary conditionals.*
   5.1. Entailment-representing conditionals are sentences of the object language that represent metalinguistic relations of entailment.
      5.1.1. On this interpretation, read 'If A then C' as encoding the metalinguistic statements 'A entails C' or 'A implies C'.
      5.1.2. The deduction theorem is useful or needed to justify such conditionals.
   5.2. Ordinary conditionals, in indicative or subjunctive mood, are sentences of the object language that represent explanatory, evidential, causal, counterfactual and perhaps other relations in worldly or epistemic states.
      5.2.1. Ordinary conditionals are context-dependent.[10]
      5.2.2. They may be considered as having a tacit ceteris-paribus clause in the antecedent.
      5.2.3. They may be considered enthymematic in that they leave unmentioned a fairly large body of tacit background theory. This background needs to be cotenable with the antecedent.
      5.2.4. These properties do not make conditionals incomplete or in any way deficient, nor are such conditionals strictly speaking false.
   5.3. There seemed to be a confluence of the logics of the two kinds of conditionals in the early works of Angell (on 'subjunctive conditionals') and McCall (on 'connexive implication'). But soon both acknowledged that their original logics were inadequate.
6. At least two kinds of connection or relevance need to be distinguished (for the purposes of connexive logic). Let us call them *content relevance*[11] and *status relevance.*
   6.1. Content Relevance means that the antecedent is related in content, meaning or topic to the consequent.
      6.1.1. In logical symbolization, this requires that the antecedent and the consequent share some variable.
      6.1.2. A strong interpretation is that the content of the consequent is contained in the content of the antecedent.
   6.2. Status relevance means that the antecedent in some sense promotes the consequent.
      6.2.1. This may mean that the state of affairs described in the antecedent brings about the state of affairs described in the consequent, or at least raises the latter's objective probability (a metaphysical relation).
      6.2.2. Alternatively, this may mean that the information conveyed by the

[9] Alternative terminologies: consequence-representing, inference-representing, implication-representing. The last term should be avoided. Priest [4, pp. 82–83, 167] considers ordinary conditionals as 'expressing laws of logic'.

[10] Examples: Gibbard's [2, p. 231] poker game, Priest's [4, p. 84] overtaking; cf. Quine's [6, p. 203] Caesar example for subjunctive conditionals.

[11] Alternative terminologies: meaning relevance, topic relevance.

antecedent brings about the acceptance of the consequent, or at least raises the latter's plausibility or subjective probability (an epistemic relation).

6.2.3. Status relevance may be positive or negative, and there is also status irrelevance. (This speaks against the equivalence of $\neg(A \rightarrow C)$ and $A \rightarrow \neg C$. The situation is different for content relevance.)

6.2.4. 'Bringing about' and 'raising' are contrastive notions, contrasting the effects of (information about) the presence of a state of affairs with the effects of (information about) its absence. The antecedent makes a difference to the 'status' of the consequent.

6.3. Conditionals as used in ordinary language hardly ever conform to the variable sharing requirement.

6.4. Conjecture: Entailment-representing conditionals tend to express content relevance, ordinary conditionals status relevance.

7. Traditionally, connexive logic has focussed mainly on entailment-representing conditionals.

7.1. This is to a great extent due to the historical roots of connexive logic in Aristotle, Boethius and Abelard.

7.2. This may also be due, to some small extent, to using the term 'implication' for the conditional, thus perpetuating Russell's 'mistake'[12] of calling the material conditional 'material implication'.

7.3. Conjecture: Accordingly, connexive logic has focussed mainly on content relevance.

8. If connexive logic is (considered as) a theory of conditionals, then it should focus on ordinary conditionals, too.

8.1. Arguably, connexive logic should therefore focus on status relevance, too.

8.2. Arguably, connexive logic should be a theory about which conditionals (and non-conditionals) can or cannot be inferred from a given body of conditional (and non-conditional) premises, not a theory about which conditionals are logical truths. There may be no first-degree conditionals that are logical truths.

9. In any case, one should take the utmost care in explaining what exactly a given connexive logical system is supposed to *represent* or *model*.

10. First case study: *(Conjunctive) Simplification* $(A \wedge B) \rightarrow A$ and $(A \wedge B) \rightarrow B$

10.1. Simplification appears to be very plausible intuitively.

10.2. But Simplification has been given up in many connexive logics (e.g., in logics proposed by Nelson, Angell, McCall, Sylvan, Priest).

10.3. Has this been an effect of logic revision, because giving up Simplification incurs minimal damage to the reference logic (conservatism)?

10.4. Why should it be given up? Can a positive philosophical motivation be provided?

11. Second case study: *Aristotle's Second Thesis*

11.1. Aristotle's Second Thesis was absolutely central for Aristotle and Abelard (and it was used by Boethius).[13]

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[12] Quine [6, p. 179].

[13] Martin [3, 379–381] calls it 'Aristotle's First Principle', and Sylvan [7, pp. 74–75] 'AR1' (for 'Aristotle's First Principle'), while Priest, Tanaka and Weber [5] simply call it 'the connexive

11.2. It is usually symbolized as $\neg((A \to C) \wedge (\neg A \to C))$. In this form, it is dual to Abelard's First Principle.
11.3. In other variants, it is dual to Unsat2, Weak Boethius's Thesis or Boethius's Thesis.[14] (In each case, the duality seems to be similar to the duality of $(A \wedge \neg A) \to C$ and $A \to (C \vee \neg C)$; it presupposes the validity of contraposition.)
11.4. Why has it never become prominent in the modern revival of connexive logic? Has this been an effect of logic revision?
11.5. Why should it not be endorsed? Can a positive philosophical motivation for or against it be provided?

(Because of time constraints, the two case studies are unlikely to be covered in the talk.)

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principle'.

[14] Essentially following Francez [1], Wansing [8] calls the dual to Boethius's Thesis a 'variation' of Boethius's Thesis. This seems misleading to me.

Connexive Counterpart Theory

Heinrich Wansing

Ruhr University Bochum, Germany,

Heinrich.Wansing@rub.de

1 Theories

The noun 'theory' has many different meanings. It is utilized to refer to things as different as, for example, research areas (such as model, proof, decision, game, or set theory), to classes of models (according to the non-statement view of theories), deductively closed sets of statements (according to the statement view of theories), or particular axiom systems with modus ponens as the sole or one of several inference rules.

Given a logic $\mathbf{L}$ understood as a pair consisting of a language $\mathcal{L}$ and a consequence relation between sets of $\mathcal{L}$-formulas and single $\mathcal{L}$-formulas, the definition of the notion of a theory, or $\mathcal{L}$-theory, that can be found in logic textbooks such as [4, p. 104] makes no reference to any particular vocabulary. A theory T is a set of (closed) formulas of $\mathcal{L}$ with the property that if an $\mathcal{L}$-formula A is derivable from T, then A belongs to T: $T \vdash_{\mathbf{L}} A$ implies $A \in T$. If $\vdash_{\mathbf{L}}$ stands for a Tarskian consequence relation, then $\varnothing \vdash_{\mathbf{L}} A$ implies $A \in T$, i.e., all theorems belong to T.

Robert Meyer and John Slaney [12] assume a formal language $\mathcal{L}$ that contains two binary connectives, $\rightarrow$ (implication) and $\wedge$ (conjunction), and a falsity constant, $\mathbf{f}$, in order to define a negation connective by setting $\sim A := A \rightarrow \mathbf{f}$. (They use '&' instead of '$\wedge$'.) An $\mathcal{L}$-theory simpliciter then is a set T of $\mathcal{L}$-formulas such that:

– If $\vdash_{\mathbf{L}} A \rightarrow B$ and $A \in T$, then $B \in T$;

– If $A \in T$ and $B \in T$, then $A \wedge B \in T$.

It is also not uncommon to define the notion of a theory by requiring that

– If $A \vdash_{\mathbf{L}} B$ and $A \in T$, then $B \in T$;

– If $A \in T$ and $B \in T$, then $A \wedge B \in T$.

Meyer and Slaney point out that not every theory in their sense is closed under modus ponens, that not every theory in the above sense contains all theorems, that the empty set is a theory simpliciter, and that not every non-trivial theory simpliciter is consistent, i.e., is such that for no formulas A, both $A \in T$ and $\sim A \in T$. Moreover, they draw a distinction between different kinds of theories. A theory T is *regular* iff it satisfies a condition that amounts to requiring that $\{A \mid \varnothing \vdash_{\mathbf{L}} A\} \subseteq T$, and it is *detached* iff it is closed under the rule

$$\text{If } B \in T \text{ and } B \rightarrow C \in T, \text{ then } C \in T.$$

They call a theory *ordinary* iff it satisfies both conditions and suppose that "[p]ractical formal theories, in mathematics or science" will in general be detached. As to regularity, they write that they see little reason to claim that such theories are all regular: there is no more compulsion for physicists or gymnasts to assert truths of logic than for logicians to learn gymnastics [12, p. 277].

Although polemics may by stimulating, in general polemics is a bad adviser. The present paper discusses Meyer and Slaney's view by looking at David Lewis's counterpart theory.

2 Connexive counterpart theory

For the present purposes, it suffices to consider counterpart theory, CT, as introduced in [6]. Lewis introduced CT as an alternative to quantified modal logic, for a criticism of this application see [2] and for additional references see the supplementary document "Counterpart-theoretic Semantics for Quantified Modal Logic" in [5]. CT is an axiomatic first-order theory. Its language, $\mathcal{L}_{\text{CT}}$, contains the identity symbol and makes use of the following primitive predicates; $\mathsf{W}(x)$ (x is a possible world), $\mathsf{I}(x, y)$ (x is in y), $\mathsf{A}(x)$ (x is actual), and $\mathsf{C}(x, y)$ (x is a counterpart of y). The quantifiers range over possible worlds and the objects existing in these worlds. The axioms of CT are:[15]

Ax1 $\forall x \forall y(\mathsf{I}(x, y) \to \mathsf{W}(y))$ (Nothing is in anything except a world.)

Ax2 $\forall x \forall y \forall z((\mathsf{I}(x, y) \land \mathsf{I}(x, z)) \to y = z)$ (Nothing is in two worlds.)

Ax3 $\forall x \forall y(\mathsf{C}(x, y) \to \exists z \mathsf{I}(x, z))$ (Whatever is a counterpart is in a world.)

Ax4 $\forall x \forall y(\mathsf{C}(x, y) \to \exists z \mathsf{I}(y, z))$ (Whatever has a counterpart is in a world.)

Ax5 $\forall x \forall y \forall z(((\mathsf{I}(x, y) \land \mathsf{I}(z, y)) \land \mathsf{C}(x, z)) \to x = z)$
(Nothing is a counterpart of anything else in its world.)

Ax6 $\forall x \forall y(\mathsf{I}(x, y) \to \mathsf{C}(x, x))$ (Anything in a world is a counterpart of itself.)

Ax7 $\exists x\, (\mathsf{W}(x) \land \forall y\, (\mathsf{I}(y, x) \leftrightarrow \mathsf{A}(y)))$ (Some world contains all and only actual things.)

Ax8 $\exists x \mathsf{A}(x)$ (Something is actual.)

The underlying logic of CT is classical first-order logic with identity. We will consider CT based on an expansion $\mathbf{QC}^{=,\neq}$ of the connexive first-order logic **QC** from [13] (see also [10]), expanded by an identity predicate, '=', and an apartness predicate '$\neq$'. The Kripke semantics for $\mathbf{QC}^{=,\neq}$ has the following support of truth ($\models^+$) and support of falsity ($\models^-$) clauses for the additional predicates:

$$
\begin{array}{lll}
\mathcal{M}, x \models^+ t_1 = t_2 & \text{iff} & I(t_1) =_x I(t_2) \\
\mathcal{M}, x \models^- t_1 = t_2 & \text{iff} & I(t_1) \neq_x I(t_2) \\
\mathcal{M}, x \models^+ t_1 \neq t_2 & \text{iff} & I(t_1) \neq_x I(t_2) \\
\mathcal{M}, x \models^- t_1 \neq t_2 & \text{iff} & I(t_1) =_x I(t_2)
\end{array}
$$

where for every state x from a model $\mathcal{M}$, $=_x$ and $\neq_x$ are binary relations on the individual domain of x, see [7]. For our discussion, the semantics of equality

[15] Lewis [6] also presents a list of principles that are assumed not to be valid.

and apartness will, however, be inessential. The logic **QC** is a (hyper)connexive variant of David Nelson's constructive logic with strong negation **QN4** [1]. The relevant aspect of $\mathbf{QC}^{=,\neq}$ is the validity of the axiom that distinguishes the standard axiomatization of **QC** from that of **QN4**:

$$\sim(A \to B) \leftrightarrow (A \to \sim B)$$

where $\sim A$ is the strong negation of A and the support of truth/falsity clauses for strong negation and the conditional in a Kripke model $\mathcal{M}$ with pre-order $\leq$ are:

$$\begin{array}{rll}
\mathcal{M}, x \models^{+} \sim A & \text{iff} & \mathcal{M}, x \models^{-} A \\
\mathcal{M}, x \models^{-} \sim A & \text{iff} & \mathcal{M}, x \models^{+} A \\
\mathcal{M}, x \models^{+} A \to B & \text{iff} & \text{for all } y \text{ with } x \leq y,\ \mathcal{M}, y \not\models^{+} A \text{ or } \mathcal{M}, y \models^{+} B \\
\mathcal{M}, x \models^{-} A \to B & \text{iff} & \text{for all } y \text{ with } x \leq y,\ \mathcal{M}, y \not\models^{+} A \text{ or } \mathcal{M}, y \models^{-} B.
\end{array}$$

3 Discussion

Two claims will be made.

1. It will be argued that the intuitive appeal of the quote from Meyer and Slaney disappears if logical pluralism is taken into consideration. According to Roy Cook [3, p. 496], substantial logical pluralism holds that given a formal language $\mathcal{L}$ and an identification of $\mathcal{L}$'s logical vocabulary, there exist distinct consequence relations $\vdash_1$ and $\vdash_2$ such that a certain correctness principle holds for the pairs $\langle \mathcal{L}, \vdash_1 \rangle$ and $\langle \mathcal{L}, \vdash_2 \rangle$. The correctness principle may be a matter of debate, but the main point to be made here is that substantial logical pluralism acknowledges that there may be at least two consequence relations over one and the same language that represent justified options for choosing between them. It may be held that the theorems of a logic are uninformative insofar as they are true in each and every model from a class of models with respect to which the logic in question is ideally sound and complete. A choice between $\langle \mathcal{L}, \vdash_1 \rangle$ and $\langle \mathcal{L}, \vdash_2 \rangle$ will, however, involve a choice between classes of models or even between kinds of classes of models. In view of the availability of such a choice, the theorems of a logic matter. If the pluralism is substantial, then so are the differences between the logics one may choose between. The available choice may give a physicist a reason to assert the theorems of $\langle \mathcal{L}, \vdash_1 \rangle$ instead of those of $\langle \mathcal{L}, \vdash_2 \rangle$ if the latter differ with respect to their sets of theorems.

2. It will be maintained that connexive counterpart theory is an apt example to illustrate that regular theories are relevant because (the theorems of) the underlying logic may have a significant impact on the meaning of the theoretical, non-logical vocabulary.

$\mathbf{QC}^{=,\neq}$ is a non-trivial negation inconsistent logic; its propositional fragment **C** is negation inconsistent already, see also [9], [14]. In CT based on $\mathbf{QC}^{=,\neq}$, CCT, additional contradictions are provable, e.g. the pair of formulas $\exists x(\sim \mathsf{A}(x) \to \mathsf{A}(x))$ and $\sim\exists x(\sim \mathsf{A}(x) \to \mathsf{A}(x))$. Moreover, in CT the axioms Ax3 and Ax4 are provably equivalent with

Ax3′ $\forall x \forall y(\mathsf{C}(x, y) \to \sim\forall z \sim \mathsf{I}(x, z))$

Ax4′ $\forall x \forall y(\mathsf{C}(x, y) \to \sim\forall z \sim \mathsf{I}(y, z))$.

In CCT, the latter are provably strongly equivalent, and hence replaceable for each other, with

Ax3∗ $\forall x \forall y {\sim}(\mathsf{C}(x, y) \rightarrow \forall z {\sim} \mathsf{I}(x, z))$

Ax4∗ $\forall x \forall y {\sim}(\mathsf{C}(x, y) \rightarrow \forall z {\sim} \mathsf{I}(y, z))$.

In CT the latter trivialize the counterpart relation in the sense that everything is a counterpart of everything else.

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List of Participants

Andrzej Indrzejczak (andrzej.indrzejczak@filhist.uni.lodz.pl)
University of Lodz, Poland

Tadao Ishii (ishii@nuis.ac.jp)
Niigata University of Information and International Studies, Japan

Tomasz Jarmużek (tomasz.jarmuzek@umk.pl)
Nicolaus Copernicus University in Toruń, Poland

Mateusz Klonowski (mateusz.klonowski@umk.pl)
Nicolaus Copernicus University in Toruń, Poland

Wolfgang Lenzen (lenzen@uos.de)
University of Osnabrück, Germany

Sara Ayhan sara.ayhan@rub.de
Ruhr University Bochum, Germany

Piotr Łukowski (piotr.lukowski@uj.edu.pl)
Jagiellonian University, Poland

Jacek Malinowski (studia.logica@gmail.com)
Polish Academy o Sciences, Poland

Satoru Niki (satoru.niki@rub.de)
Ruhr University Bochum, Germany

Grigory Olkhovikov (grigory.olkhovikov@rub.de)
Ruhr University Bochum, Germany

Francesco Paoli (paoli@unica.it)
University of Cagliari, Italy

Hans Rott hans.rott@ur.de
University of Regensburg, Germany

Heinrich Wansing (Heinrich.Wansing@rub.de)
Ruhr University Bochum, Germany

Michał Zawidzki (michal.zawidzki@filhist.uni.lodz.pl)
University of Lodz, Poland

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