Non-Rigid Designators in Epistemic and Temporal Free Description Logics

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based on joint work with:

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Introduction

Referring expressions (REs)

Noun phrases that can **refer** to a **single object** in a context

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Noun phrases that can **refer** to a **single object** in a context

- **Individual names**: 'KR23', 'KR24', 'François-Marie Arouet', 'Voltaire', 'Clark Kent', 'Superman', ...
- **Definite descriptions**: 'the next KR conference', 'the most famous French thinker alive', 'the love interest of Lois Lane', ...

Introduction

Referring expressions (REs)

Noun phrases that can **refer** to a **single object** in a context

- Serve as **meaningful** and **flexible object descriptors** in natural languages for human communication
- Mitigate the **obscurity** of **object identifiers** in information and knowledge base management systems

Introduction

Description Logics (DLs)

(Typically) **decidable fragments** of FOL used for **knowledge representation** and **reasoning** tasks

Epistemic and temporal DLs

Extensions of DLs with **modal operators** to reason about agents' **epistemic states** and **temporal evolution** of objects, respectively

Motivations and Goals

REEs in epistemic and temporal DLs

Syntax

nominals with **individual** names, $\{a\}$,
and **definite description** terms, $\{\iota C\}$

Semantics

denoting vs. **non-denoting** terms at a world
rigid vs. **non-rigid** terms across worlds

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Denoting	Non-Denoting
'KR23', 'the General Chair of KR23', ...	'KR19' (did not exist), 'the Program Chair of KR23' (more than one), 'the deadline extension for KR23' (none), 'KR24' (does not exist yet), ...

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Rigid	**Non-Rigid**
'Rhodes', 'Paper$X$', 'Hanoi', 'Paper$Y$', ...	'the KR location', 'the winner of Best Paper Award at KR', ...

Motivations and Goals

Satisfiability in epistemic and temporal DLs with REs

Study **decidability** and **complexity** of **satisfiability problems**

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- **Temporal** settings: reasoning about **dynamic values**
- **Epistemic** settings: reasoning about (**un**)**known identities**

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N.B.

Undecidability is round the corner!

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Study **decidability** and **complexity** of **satisfiability problems**

- **Temporal** settings: reasoning about **dynamic values**
- **Epistemic** settings: reasoning about (**un**)**known identities**

N.B.

Undecidability is round the corner!

- **Minsky machine** encoding via **non-rigid designation/counting up to one** and **temporal structures** interactions
- **Decidability regained** on **epistemic structures** (via **quasi-models**, similarly to **product S5 $\times$ S5**)

Free DLs with Definite Descriptions - The Non-Modal Case

[AMOW20], [AMOW21]

Free DLs with Definite Descriptions - The Non-Modal Case

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Languages

- Extend standard languages to include both **individual names** a and **definite descriptions** ιC ('the C') as **terms**
- Generalise classical semantics with **partial interpretations**: total on concept/role names and **partial** on **individual names**
  - $\rightsquigarrow$ **Free logic** semantics for non-denoting terms [B02, NKR20]

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Reasoning tasks

- **Ontology satisfiability** and **entailment**
- $(\mathcal{L}, \mathcal{L}_R)$ **RE existence**: given a pair of **logics** $(\mathcal{L}, \mathcal{L}_R)$, decide, for a **background** $\mathcal{L}$ **ontology** $\mathcal{O}$, an **individual name** a, and a **signature** Σ, whether there exists an **RE** $\mathcal{L}_R(\Sigma)$ **concept** that **describes** a **under** $\mathcal{O}$, i.e., such that $\mathcal{O} \models \{a\} \equiv C$

Free Description Logics - Syntax $\mathcal{ALCO}_u^\iota$

Definition (Terms, Concepts, Axioms)

- **Terms**: $\tau ::= \boldsymbol{a} \mid \boldsymbol{\iota C}$
- **Concepts**: $C ::= A \mid \boldsymbol{\{\tau\}} \mid \neg C \mid C \sqcap C \mid \exists r.C \mid \boldsymbol{\exists u.C}$
- **Axioms**: $C \sqsubseteq C \mid C(\tau) \mid r(\tau, \tau)$
- **Ontology**: finite set of axioms

Ontology Example

$$\{\text{kr19}\} \sqsubseteq \bot \qquad \top(\text{kr21}) \qquad \{\text{kr20}\} \sqcap \{\text{kr21}\} \sqsubseteq \bot$$

$$\{\text{kr20}\} \sqcup \{\text{kr21}\} \equiv \text{KRConf} \sqcap \exists \text{hasLoc}.\text{VirtualLoc} \sqcap \forall \text{hasLoc}.\text{VirtualLoc}$$

$$\exists \text{isProgramChairOf}.\{\text{kr20}\}(\iota \exists \text{isGeneralChairOf}.\{\text{kr21}\})$$

$$\exists \text{isProgramChairOf}.\{\text{kr21}\} \sqsubseteq \neg\{\iota \exists \text{isProgramChairOf}.\{\text{kr21}\}\}$$

$$\exists \text{isProgramChairOf}.\{\text{kr21}\} \sqsubseteq \exists \text{reportsTo}.\{\iota \exists \text{isGeneralChairOf}.\{\text{kr21}\}\}$$

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Free Description Logics - Semantics I

Definition (Partial interpretation)

$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, with $\Delta^{\mathcal{I}} \neq \emptyset$ (*domain* of $\mathcal{I}$), and $\cdot^{\mathcal{I}}$ function mapping

- concept names A to $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- role names r to $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- universal role u to $u^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- individual names a in a **subset of individual names** to $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

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Definition (Total interpretation)

$\mathcal{I}$ is a **total interpretation** interpretation when $\cdot^{\mathcal{I}}$ is defined as above, except that it maps **every** $a \in \mathsf{N_I}$ to an element of $\Delta^{\mathcal{I}}$

Free Description Logics - Semantics II

Definition (Value of a term)

$\tau^{\mathcal{I}}$ is $a^{\mathcal{I}}$, if $\tau = a$, while for $\tau = \iota C$:

$$(\iota C)^{\mathcal{I}} = \begin{cases} d, & \text{if } C^{\mathcal{I}} = \{d\}, \text{ for some } d \in \Delta^{\mathcal{I}} \\ \textbf{undefined}, & \text{otherwise} \end{cases}$$

τ **denotes** in $\mathcal{I}$ iff $\tau^{\mathcal{I}} = d$, for a $d \in \Delta^{\mathcal{I}}$

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Definition (Extension of a concept)

Extension $C^{\mathcal{I}}$ of a concept C in $\mathcal{I}$:

$$\neg C^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \qquad (C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$$

$$(\exists r.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{there exists } e \in C^{\mathcal{I}} : (d, e) \in r^{\mathcal{I}}\}$$

$$(\exists u.C)^{\mathcal{I}} = \{d \in \Delta^{\mathcal{I}} \mid \text{there exists } e \in C^{\mathcal{I}} : (d, e) \in u^{\mathcal{I}}\}$$

$$\{\tau\}^{\mathcal{I}} = \begin{cases} \{\tau^{\mathcal{I}}\}, & \text{if } \tau \text{ \textbf{denotes} in } \mathcal{I} \\ \emptyset, & \text{otherwise} \end{cases}$$

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Satisfiability on **total** interpretations can be polynomial-time **reduced** to satisfiability on **partial** interpretations

For each individual name a in $\mathcal{O}$, add conjuncts $\top \sqsubseteq \exists u.\{a\}$

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Remark 3

Assertions, i.e., $C(\tau)$ or $r(\tau, \tau')$, are **syntactic sugar**

- $C(\tau) \quad \rightsquigarrow \top \sqsubseteq \exists u.\{\tau\}$, $\{\tau\} \sqsubseteq C$
- $r(\tau_1, \tau_2) \rightsquigarrow \top \sqsubseteq \exists u.\{\tau_1\}$, $\{\tau_1\} \sqsubseteq \exists r.\{\tau_2\}$

Reasoning in $\mathcal{ALCO}_u^\iota$

Remark

Concept inclusions (CIs) $C \sqsubseteq D$ assumed, w.l.o.g. for satisfiability, to be in **normal form**, that is, of the form: $E \sqsubseteq F$, with E, F $\mathcal{ALC}$ concepts, $\{\tau\} \sqsubseteq A$, or $A \sqsubseteq \{\tau\}$, with A concept name and τ either **individual name** or of the form ιB, with B concept name

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Remove the ι: Reduce $\mathcal{ALCO}_u^\iota$ to $\mathcal{ALCO}_u$ ontology satisfiability

- $\{\iota B\}^\dagger = B \sqcap \forall u.(B \Rightarrow \{a_{\iota B}\})$,
- $\{b\}^\dagger = A_b \sqcap \forall u.(A_b \Rightarrow \{a_b\})$,
- $(\{\tau\} \sqsubseteq A)^\dagger / (A \sqsubseteq \{\tau\})^\dagger = (\{\tau\}^\dagger \sqsubseteq A)/(A \sqsubseteq \{\tau\}^\dagger)$, and $\{\tau\}^+ \sqsubseteq \forall u.(\{a_\tau\} \Rightarrow \{\tau\}^+)$, where $\{\iota B\}^+ = B$, $\{b\}^+ = A_b$

Theorem

*$\mathcal{ALCO}_u^\iota$ **ontology satisfiability** (on **partial** and **total** interpretations) is **ExpTime-complete***

Reasoning in $\mathcal{ELO}_u^\iota$

Adapt the **completion algorithm** for $\mathcal{ELO}$ ontologies [BBL05]

- add to classification graph a copy of each concept name in $\mathcal{O}$
- remove it only if the concept has 1 element in any model of $\mathcal{O}$

Theorem

Entailment *in* $\mathcal{ELO}_u^\iota$ *(on partial and total interpretations) is* **PTime-complete**

$\mathcal{ALCO}_u^\iota$ Bisimulations and Expressive Power

Definition ($\mathcal{ALCO}_u^\iota(\Sigma)$ bisimulation)

$Z \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{J}$ $\mathcal{ALCO}_u^\iota(\Sigma)$ **bisimulation** between $\mathcal{I}$ and $\mathcal{J}$

(bisim) Z $\mathcal{ALCO}(\Sigma)$ bisimulation

(total) $\Delta^\mathcal{I}$ domain and $\Delta^\mathcal{J}$ range of Z

(ι) $\exists d' \in \Delta^\mathcal{I}$ s.t. $d \neq d'$ and $(\mathcal{I}, d) \sim_\Sigma^{\mathcal{ALCO}} (\mathcal{I}, d') \Leftrightarrow$
$\exists e' \in \Delta^\mathcal{J}$ s.t. $e' \neq e$ and $(\mathcal{J}, e) \sim_\Sigma^{\mathcal{ALCO}} (\mathcal{J}, e')$

Definition (FOL standard translation)

$$\pi_x(A) = A(x) \qquad \pi_x(\neg C) = \neg\pi_x(C) \qquad \pi_x(C \sqcap D) = (\pi_x(C) \wedge \pi_x(D))$$

$$\pi_x(\exists r.C) = \exists y(r(x, y) \wedge \pi_y(C)) \qquad \pi_x(\exists u.C) = \exists x \pi_x(C)$$

$$\pi_x(\{a\}) = x = a$$

$$\pi_x(\{\iota C\}) = \exists x \pi_x(C) \wedge \forall x \forall y(\pi_x(C) \wedge \pi_y(C) \rightarrow x = y) \wedge \forall y(\pi_y(C) \rightarrow x = y)$$

$\mathcal{ALCO}_u^\iota$ Bisimulations and Expressive Power

Theorem

For a signature Σ and an FOL formula $\varphi(x)$ such that $\Sigma_{\varphi(x)} \subseteq \Sigma$, the following are equivalent, on partial interpretations [a]*:*

1. *there exists an $\mathcal{ALCO}_u^\iota(\Sigma)$ concept C such that $\pi_x(C)$ is logically equivalent to $\varphi(x)$*
2. *$\varphi(x)$ is invariant under $\sim_\Sigma^{\mathcal{ALCO}_u^\iota}$*

[a] FOL partial interpretation semantics naturally extends the DL one above

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[a] FOL partial interpretation semantics naturally extends the DL one above

$\mathcal{ALCO}_u^\iota$ is the **fragment** of **FOL** on **partial interpretations** that is **invariant under $\mathcal{ALCO}_u^\iota$-bisimulations**

Referring Expression Existence

Definition (($\mathcal{L}, \mathcal{L}_R$) RE existence)

Given a pair $(\mathcal{L}, \mathcal{L}_R)$ of **logics**, **decide**, for an $\mathcal{L}$ **ontology** $\mathcal{O}$, an **individual name** a, and a **signature** Σ, whether **there exists** an $\mathcal{L}_R(\Sigma)$ **concept** C such that $\mathcal{O} \models \{a\} \equiv C$

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$\mathcal{L}$ **RE existence**, if $\mathcal{L} = \mathcal{L}_R$

Referring Expression Existence

Theorem

On partial and total interpretations:

1. $(\mathcal{ALCO}_u^\iota, \mathbf{FO})$ *RE existence is* **ExpTime-complete**
2. $(\mathcal{ELO}_u^\iota, \mathbf{FO})$ *RE existence is* **in PTime**
3. $\mathcal{ALCO}_u^\iota$ *RE existence is* **2ExpTime-complete**
4. $(\mathcal{ALCO}_u^\iota, \mathcal{ELO}_u^\iota)$ *RE existence is* **undecidable**
5. $\mathcal{ELO}_u^\iota$ *RE existence is* **in PTime**, *for* **individuals** *that* **denote** *w.r.t. the ontology*

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1.-2. FO projective Beth definability property on total/partial ints. + $\mathcal{ALCO}_u^\iota$ $\textsc{ExpTime}/\mathcal{ELO}_u^\iota$ $\textsc{PTime}$ reasoning upper bound

3. Bisimulation-based characterisation of $\mathcal{ALCO}_u^\iota$ RE existence + mosaic-based technique (upper bound) / exponential-space bounded Alternating Turing Machines (lower bound) [AJMOW21]
4. Undecidability proof for CQ inseparability of $\mathcal{ALC}$ KBs [BLRWZ19]
5. $\mathcal{ELO}_u(\Sigma)$ REs existence + simulation-based characterisation of $\mathcal{ELO}_u$ RE existence

Excursus – Free DLs with Dual-Domain Semantics

Definition ($\mathcal{ALCO}^{\iota *}$ [NKR20])

- $\mathcal{ALCO}^{\iota *}$ concepts

$$\mathcal{ALCO}^{\iota} \text{ concepts} + \mathbf{T} \text{ (existing objects)}$$

- $\mathcal{ALCO}^{\iota *}$ formulas

$$\varphi ::= C \sqsubseteq D \mid C(\tau) \mid r(\tau_1, \tau_2) \mid \tau_1 = \tau_2 \mid \neg(\varphi) \mid (\varphi \land \varphi)$$

Excursus – Free DLs with Dual-Domain Semantics

Definition (Dual-domain interpretation)

$I = (\Delta^I, \mathfrak{d}^I, \cdot^I)$

- Δ^I non-empty set, outer domain of I
- $\mathfrak{d}^I \subset \Delta^I$ (possibly empty) set, inner domain of I
- $\cdot^I$ (standard) interpretation function in Δ^I

$$(\iota C)^I = \begin{cases} d, & \text{if } \mathfrak{d}^I \cap C^I = \{d\} \\ d_{\iota C}, & \text{with } d_{\iota C} \in \Delta^I \setminus \mathfrak{d}^I \\ & \text{arbitrary, otherwise} \end{cases}$$

$$\mathbf{T}^I = \mathfrak{d}^I, \quad \{\tau\}^I = \{\tau^I\},$$

$$(\exists r.C)^I = \{d \in \Delta^I \mid \exists e \in \mathfrak{d}^I \cap C^I : (d,e) \in r^I\}$$

Definition (Positive (+) and negative (−) semantics)

$$I \models^+ C(\tau) \quad \text{iff} \quad \tau^I \in C^I$$

$$I \models^- C(\tau) \quad \text{iff} \quad \tau^I \in \mathfrak{d}^I \text{ and } \tau^I \in C^I$$

Excursus – Free DLs with Dual-Domain Semantics

Theorem

$\mathcal{ALCO}^{\iota *}$ **formula satisfiability** *on* **dual domain interpretations** *under* **positive** *or* **negative** *semantics is polynomial time* **reducible** *to* $\mathcal{ALCO}_u^{\iota}$ **ontology satisfiability** *on* **partial interpretations**

Epistemic & Temporal Extensions of Free DLs

[AM23]

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Syntax

Extensions of standard DLs with both **RE** and **modal** constructors

- **Individual names** a and **definite descriptions** ιC ('the C') both **terms** of the language
- **Modal** and **temporal** operators representing **knowledge**/**belief** states and **temporal** evolution

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Modal structures of **partial interpretations** that are total on concept/role names and **partial** on **individual names**

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Semantics

Modal structures of **partial interpretations** that are total on concept/role names and **partial** on **individual names**

- **Epistemic structures**: **equivalence classes** of partial interpretations
- **Temporal structures**: (finite or infinite) **sequences** of partial interpretations

Epistemic Free DL Language

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Definition (Terms, Concepts, Axioms)

$\mathbf{S5}_{\mathcal{ALCO}_u^{\iota}}$

- **Terms**: $\tau ::= a \mid \iota C$
- **Concepts**: $C ::= A \mid \{\tau\} \mid \neg C \mid C \sqcap C \mid \exists r.C \mid \exists u.C \mid \Diamond \boldsymbol{C}$
- **Axioms**[a]: $C \sqsubseteq C \mid C(\tau) \mid r(\tau, \tau)$
- **Formulas**[a]: Boolean and modal axiom combinations

[a] Assertions and formulas are syntactic sugar due to *universal role* u

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$\Diamond C \rightsquigarrow$ "objects that are (**epistemically**) **conceivable** as C"

$\neg\Diamond\neg C := \Box C \rightsquigarrow$ "objects that are **known** to be C"

Temporal Free DL Language

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Definition (Terms, Concepts, Axioms)

LTL$_{\mathcal{ALCO}_u^\iota}$

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$\bot\,\mathcal{U}\,C := \bigcirc C \rightsquigarrow$ "objects that **tomorrow** will be C"

$\top\,\mathcal{U}\,C := \Diamond C \rightsquigarrow$ "objects that will **eventually** be C"

$\neg\Diamond\neg C := \Box C \rightsquigarrow$ "objects that will **always** be C"

$+$ "reflexive" (i.e., including the present) operators $\Diamond^+$, $\Box^+$

Epistemic Free DL Semantics

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Definition (Epistemic frame)

$\mathfrak{F} = (W, \sim)$, with:

- W non-empty set of **worlds**
- $\sim \subseteq W \times W$ **equivalence relation** on W

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Definition (Partial epistemic interpretation)

$\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, with:

- $\mathfrak{F}$ **epistemic frame** of $\mathfrak{M}$
- Δ non-empty **domain** of $\mathfrak{M}$ (**constant domain assumption**)
- $\mathcal{I}$ function mapping each $w \in W$ to **partial interpretation** $\mathcal{I}_w$

$\mathfrak{M}$ is a **total epistemic interpretation** iff every $\mathcal{I}_w$ is total

Denotation and Rigidity

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Definition (Denoting individual name)

An individual name $a \in \mathsf{N_I}$:

- **denotes in $\mathcal{I}_w$** iff $a^{\mathcal{I}_w}$ is defined
- **denotes in $\mathfrak{M}$** iff a denotes in $\mathcal{I}_w$, for some $w \in W$
- is a **ghost** in $\mathfrak{M}$ iff a does not denote in $\mathfrak{M}$

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Definition (Rigid designator assumption)

$\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, with $\mathfrak{F} = (W, \sim)$, satisfies the **rigid designator assumption** (**RDA**) iff, for every individual name $a \in \mathsf{N_I}$ and every world $w, v \in W$, the following condition holds:

$a^{\mathcal{I}_w}$ is **defined** $\Rightarrow a^{\mathcal{I}_w} = a^{\mathcal{I}_v}$, i.e., a is a **rigid designator**

Epistemic Interpretation of Terms and Concepts

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Definition (Value of a term in a world)

$\tau^{\mathcal{I}_w}$ is $a^{\mathcal{I}_w}$, if $\tau = a$, while for $\tau = \iota C$:

$$(\iota C)^{\mathcal{I}_w} = \begin{cases} \boldsymbol{d}, & \text{if } \boldsymbol{C^{\mathcal{I}_w} = \{d\}},\ \text{for some } d \in \Delta \\ \textbf{undefined}, & \text{otherwise} \end{cases}$$

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Definition (Extension & satisfaction of a concept in a world)

$C^{\mathcal{I}_w}$ given as usual, with the following additions:

$$\Diamond \boldsymbol{C^{\mathcal{I}_w}} = \{d \in \Delta \mid \exists v \in W, w \sim v \colon d \in C^{\mathcal{I}_v}\},$$

$$\boldsymbol{\{\tau\}^{\mathcal{I}_w}} = \begin{cases} \boldsymbol{\{\tau^{\mathcal{I}_w}\}}, & \text{if } \tau \textbf{ denotes} \text{ in } \mathcal{I}_w, \\ \emptyset, & \text{otherwise} \end{cases}$$

C is **satisfied at** w **of** $\mathfrak{M}$ if $C^{\mathcal{I}_w} \neq \emptyset$.

Epistemic Formula (Partial) Satisfiability

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Definition ($\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ formula satisfaction)

$\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ formula φ **satisfaction at w of $\mathfrak{M}$**, $\mathfrak{M}, w \models \varphi$:

- $\mathfrak{M}, w \models C(\tau)$ iff τ **denotes** in $\mathcal{I}_w$ & $\tau^{\mathcal{I}_w} \in C^{\mathcal{I}_w}$,
- $\mathfrak{M}, w \models r(\tau_1, \tau_2)$ iff τ_1, τ_2 **denotes** in $\mathcal{I}_w$ & $(\tau_1^{\mathcal{I}_w}, \tau_2^{\mathcal{I}_w}) \in r^{\mathcal{I}_w}$,
- $\mathfrak{M}, w \models C \sqsubseteq D$ iff $C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}$,
- $\boldsymbol{\mathfrak{M}, w \models \Diamond \psi}$ iff $\exists v \in W, w \sim v \colon \mathfrak{M}, v \models \psi$,
- $+$ usual Boolean clauses

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- $+$ usual Boolean clauses

Definition ($\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ partial/total satisfiability)

An **$\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ formula** φ is:

- **satisfied in $\mathfrak{M}$** if there is a world w in $\mathfrak{M}$ such that $\mathfrak{M}, w \models \varphi$
- **partial** (resp., **total**) **satisfiable** if there is a partial (total) modal interpretation $\mathfrak{M}$ such that φ is satisfied in $\mathfrak{M}$

Temporal Free DL Semantics

Definition (Temporal frame & Partial temporal interpretation)

- $\mathfrak{F} = (\mathbb{N}, <)$: $\mathbb{N}$ set of **natural numbers**; $<$ linear order on $\mathbb{N}$
- $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, as in epistemic case

Temporal Interpretation of Concepts and Formulas

Definition (LTL$_{\mathcal{ALCO}_u^\iota}$ concept & formula satisfaction)

The **value** of an **LTL**$_{\mathcal{ALCO}_u^\iota}$ **term** τ, the **extension** of an **LTL**$_{\mathcal{ALCO}_u^\iota}$ **concept** C, and the **satisfaction** of a **LTL**$_{\mathcal{ALCO}_u^\iota}$ **formula** φ at **instant** t of partial temporal interpretation $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, are defined similarly to the epistemic case, with the clauses:

$$\boldsymbol{(C\,\mathcal{U}\,D)^{\mathcal{I}_t}} = \{d \in \Delta \mid \exists u > t \colon d \in D^{\mathcal{I}_u} \;\&\; \forall v \in (t, u) \colon d \in C^{\mathcal{I}_v}\}$$

$$\boldsymbol{\mathfrak{M}, t \models \varphi\,\mathcal{U}\,\psi} \text{ iff } \exists u > t \colon \mathfrak{M}, u \models \psi \;\&\; \forall v \in (t, u) \colon \mathfrak{M}, v \models \varphi$$

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Definition ($\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$ partial/total satisfiability)

An $\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$ formula φ/concept C is **partial** (resp., **total**) **satisfiable** iff φ/C is satisfied at instant 0 in some partial (resp., total) temporal interpretation $\mathfrak{M}$

Partial vs. Total Interpretations

Remarks

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- **Dropping the RDA** is the most **general** assumption: **rigid designators** can be **enforced** by the CI:

$$\Diamond^{+}\{a\} \sqsubseteq \Box^{+}\{a\}$$

Partial vs. Total Interpretations

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- **Dropping the RDA** is the most **general** assumption: **rigid designators** can be **enforced** by the CI:

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- **Partial** interpretations **generalise** the **total** (standard) ones: individual names can be forced to denote at some/every world

$$\top \sqsubseteq \Diamond^+\exists u.\{a\} \quad / \quad \top \sqsubseteq \Box^+\exists u.\{a\}$$

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$$\top \sqsubseteq \Diamond^{+}\exists u.\{a\} \quad / \quad \top \sqsubseteq \Box^{+}\exists u.\{a\}$$

- **Interesting satisfiability** phenomena **without the RDA**, e.g.

$$(\{a\} \sqsubseteq \Box C) \wedge \Diamond(\{a\} \sqsubseteq \neg C)$$

is **satisfiable without the RDA** (when a is interpreted differently across worlds) and **unsatisfiable with the RDA**

Epistemic Scenario

Example

Characters

- Clark (clark), Lois (lois), Superman (superman)
- $\Box$ ("Lois knows")

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Story

- In the actual scenario, Clark *is* Superman, but for Lois there is an epistemically conceivable alternative in which he *is not*

  $$\mathfrak{M}, w \models \{\text{clark}\} \equiv \{\text{superman}\} \;\&\; \mathfrak{M}, v \not\models \{\text{clark}\} \equiv \{\text{superman}\}$$

- Lois knows that Superman is the hero that saves her

  $$\mathfrak{M}, w \models \Box(\{\text{superman}\} \equiv \{\iota(\text{Hero} \sqcap \exists \text{saves}.\{\text{lois}\})\})$$

- Lois loves who she knows to be the hero that saves her

  $$\mathfrak{M}, w \models \{\text{lois}\} \sqsubseteq \exists \text{loves}.\Box\{\iota(\text{Hero} \sqcap \exists \text{saves}.\{\text{lois}\})\}$$

- Lois actually loves Clark without even realising it

  $$\mathfrak{M}, w \models \{\text{lois}\} \sqsubseteq \exists \text{loves}.\{\text{clark}\} \wedge \neg\Box(\{\text{lois}\} \sqsubseteq \exists \text{loves}.\{\text{clark}\})$$

Temporal Scenario

Example

Characters

- KR Conference (kr), KR23, KR24 (kr23, kr24), a Program Chair of KR ($\exists$isPrgChr.$\{$kr$\}$), the General Chair of KR ($\iota\exists$isGenChr.$\{$kr$\}$), a PC Member of KR ($\exists$isPCMbr.$\{$kr$\}$), the Proceedings of KR23 ($\iota\exists$isProcOf.$\{$kr23$\}$), ...
- $\bigcirc$ ("next year"), $\Diamond^+$ ("now or eventually"), $\Box^+$ ("now and forever"), ...

Temporal Scenario

Example

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- $\bigcirc$ ("next year"), $\Diamond^+$ ("now or eventually"), $\Box^+$ ("now and forever"), ...

Story

- KR24 is a rigid designator

$$\mathfrak{M}, t \models \Diamond^+\{\mathsf{kr24}\} \sqsubseteq \Box^+\{\mathsf{kr24}\}$$

- KR24 is the upcoming KR Conference, but there will be more (hopefully)

$$\mathfrak{M}, t \models \exists u.\{\mathsf{kr24}\} \wedge \{\mathsf{kr24}\} \sqsubseteq \bigcirc\{\mathsf{kr}\} \ \& \ \mathfrak{M}, t \not\models \Diamond^+\{\mathsf{kr}\} \sqsubseteq \bigcirc\{\mathsf{kr}\}$$

- However, KR24 will never come back (as the current KR conference)

$$\mathfrak{M}, t \models \Box^+(\{\mathsf{kr24}\} \sqsubseteq \bigcirc\Box\neg\{\mathsf{kr}\})$$

- Whoever is a Program Chair of KR always becomes either the General Chair or a PC Member of KR next year

$$\mathfrak{M}, t \models \Box^+(\exists \mathsf{isPrgChr}.\{\mathsf{kr}\} \sqsubseteq \{\iota\exists \mathsf{isGenChr}.\bigcirc\{\mathsf{kr}\}\} \sqcup \exists \mathsf{isPCMbr}.\bigcirc\{\mathsf{kr}\})$$

Reasoning in Epistemic Free DLs

- It is known that **without definite descriptions** (ιC) the logic $\mathbf{S5}_{\mathcal{ALCO}_u}$ is $\textsc{NExpTime}$-complete (see [GKWZ03])
- We proved that the **addition** of **definite descriptions** does not increase the complexity, even without RDA [AM23].

Theorem

Partial $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ satisfiability without RDA *is* **NExpTime-complete**

- **Decidability** proof based on a **non-deterministic** procedure guessing **quasimodels** of **exponential size** (by adapting the the proof for $\mathbf{S5} \times \mathbf{S5}$ in [GKWZ03])

Reasoning in Temporal Free DLs

- It is known that **without definite descriptions** (ιC) and with **nominals under the RDA** the logic **LTL**$_{\mathcal{ALCO}_u}$ is ExpSpace-complete (see [GKWZ03])
- We proved that the logic becomes **undecidable** by either adding **nominals without RDA** or **definite descriptions** [AM23]

Theorem

The following are **undecidable***:*

- **total** *(hence* **partial***)* **LTL**$_{\mathcal{ALCO}_u}$ **satisfiability without RDA**
- **total** *(hence* **partial***)* **LTL**$_{\mathcal{ALCO}_u^{\iota}}$ **satisfiability with RDA**

- **Undecidability** proof based on **2-counter Minsky machine** encoding via **non-rigid individual names/definite descriptions** (for **+1/0-test & -1 register operations**) and **time points** (for **computation steps**)

Ongoing Work

Epistemic Free DLs

- **Decidability** and **complexity** results for **multi-modal** cases: $\mathbf{K}^n, \mathbf{KD45}^n, \mathbf{S5}^n$
- **Undecidability** results for **universal modality** $\boxed{u}$, **common knowledge** C, or **subsumption under global ontology**

Theorem (unpublished)

1. *Let $L \in \{\mathbf{K}^n, \mathbf{KD45}^n, \mathbf{S5}^n\}$, with $n \geq 1$. Then satisfiability without the RDA in $L_{\mathcal{ALCO}^{\iota}_{u}}$ is NExpTime-complete*
2. *Let $L \in \{\mathbf{K}^n_{\boxed{u}}, \mathbf{KD45}^m_{\boxed{u}}, \mathbf{S5}^m_{\boxed{u}}\}$, with $n \geq 1$ and $m \geq 2$. Then satisfiability without the RDA in $L_{\mathcal{ALCO}^{\iota}_{u}}$ is undecidable*
3. *Let $L \in \{\mathbf{KD45}^m_C, \mathbf{S5}^m_C\}$, with $m \geq 2$. Then satisfiability without the RDA in $L_{\mathcal{ALCO}^{\iota}_{u}}$ is undecidable*
4. *Subsumption under global ontology without the RDA in $\mathbf{K}_{\mathcal{ALCO}^{\iota}_{u}}$ is undecidable*

Ongoing Work

Temporal Free DLs

- Results on **reasoning** both over **finite** (**$\mathbf{LTL}^{f}$**) and over **infinite** (**LTL**) traces, and with **box** ($\Box$) and **next** ($\bigcirc$) **operators only**
- **Without the RDA**, **undecidability is pervasive**, already affecting **$\mathbf{LTL}_{\mathcal{ALCO}}$ satisfiability** and **$\mathbf{LTL}_{\mathcal{ELO}}$ subsumption** with **global axioms alone**

Theorem (unpublished)

The following problems are **undecidable***:*

1. **$\mathbf{LTL}^{f}_{\mathcal{ALCO}}$** *satisfiability without the RDA*
2. **$\mathbf{LTL}_{\mathcal{ALCO}}$** *satisfiability without the RDA (even with global axioms alone)*
3. **$\mathbf{LTL}^{f}_{\mathcal{ELO}}$** *subsumption without the RDA*
4. **$\mathbf{LTL}_{\mathcal{ELO}}$** *subsumption without the RDA (even with global axioms alone).*

Future Work

- **Epistemic** Free DLs
  - **Less expressive epistemic DLs**, based e.g. on $\mathcal{ELO}_u^\iota$
  - Connections with recent **standpoint DLs** for multi-perspective knowledge representation
  - **Non-normal modal DLs** with definite descriptions, to avoid **logical omniscience** of normal systems
- **Temporal** Free DLs
  - **Tame undecidability**: further **restrictions** on **temporal** or **DL operators**? (e.g., temporal operators on formulas only)
- **RE existence** (+ related **interpolation** & **definability** properties) in **modal** extensions of free DLs with definite descriptions

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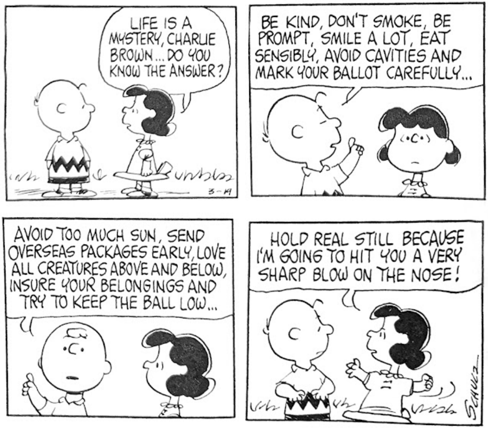
Questions



Thank you