DANIEL WATERMAN:

Mathematician, Mentor, Friend

Pamela Pierce
The College of Wooster

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Institut Matematyki
Uniwersytet Szczeciński

Who is Dan Waterman?

- Dan was one of the early members of the Summer Symposium group.
- Early Summer Symposia:
  1. Western Illinois University(1978)
  2. U. of Wisconsin, Milwaukee(1979)
  3. Michigan State University(1980)
  4. **Syracuse University** (1981–hosted by Dan). Invited speakers included B. Thomson and T. Nishiura. The Andy went to Casper Goffman.

Participation in Summer Symposia

- Dan attended the symposia pretty regularly.
- In 1993 (Carleton College, MN), Dan invited a thesis student to attend the meeting (Pam).
- Dan was an invited speaker.
- Dan won the Andy that year!
- He had a witness.

Dan's Life

- Dan grew up in Brooklyn
- Graduated high School at age 15
- Enrolled in Brooklyn College
- He intended to pursue medicine or dentistry.
- Took Advanced Calculus with Roger Johnson and "it changed my life completely."
- Graduated college in 1947.

Johns Hopkins

- Dan began studying at Johns Hopkins University for graduate school
- "B.L. Van der Waerden was the best teacher I ever had."(he was a visitor there)
- Aurel Wintner, who was intimidating to most students, took a liking to Dan, and gave him problems to work on.
- Dan was very impressed as he read Zygmund's *Trigonometric Series*, and wanted to work with him.
- Dan applied to Chicago and left Hopkins after a year.

Chicago

Zygmund: "Mr. Waterman, I have the impression that you are, how to say, somewhat lazy. If that is the case, then you cannot work with me. "

So Dan made a point of sitting at a very visible table in the mathematics library every day, to convince Zygmund that he was not lazy.

Dan was able to extend a result of Zygmund's from L^1 to $L^p, p > 1$, and thought that might be his dissertation.

Zygmund: "Do you have children?" Dan: "NO."

Zygmund: "Are you married?" Dan: "NO."

Zygmund: "Well, in that case I am going to keep you around for a while and see what I can get out of you."

Chicago

- Dan worked for several years with Zygmund (one semester with Graves).
- In the last year and a half of grad school Dan worked as an associate in the Cowles Commission for Research in Economics. Herstein was there too and they became friends.
- The commission was moving to Yale and Dan was invited to go with them, but he declined.
- He was offered a Fullbright grant to the University of Vienna and went there in 1952.
- While Dan was in Vienna, Zygmund "arranged two offers" for him, and Dan chose to go to Purdue.

Purdue

- Purdue hired 12 instructors that year. "Many of us did not come from places with the standards of dress and decorum that (the chair) was trying to maintain."
- Dan befriended Michael Golomb, Lamberto Cesari, Casper Goffman, Robert Zink.
- Cas Goffman's influence "altered my view of mathematics considerably."
- While at Notre Dame, Paul Erdős would frequently visit the Golombs on the weekends. he would come to Dan with a stack of $ 5 bills, and have Dan write checks to various charities, particularly to Native American groups. Dan learned from Erdős about the hardships faced by these groups, and this influenced Dan's charitable giving in the years that followed.

Purdue

- Dan's research blossomed while at Purdue. He wrote several papers, started a strong mathematical relationship with Goffman, and worked with 2 doctoral students.
- Eventually, there was an altercation with the dean. Dan was teaching an undergraduate real analysis course, and some under-prepared graduate students were also enrolled. The Dean wanted Dan to award the graduate students a higher grade for comparable achievement, and Dan refused.
- The Dean called Dan into his office and said: "Dan'l, Purdue isn't big enough for the both of us. I guess you know what that means." Dan said it was like a scene from an old western movie.
- Dan did not have to look for another position. His Purdue friends spoke to M. Marden at University of Wisconsin, Milwaukee, and Dan was hired immediately.

Milwaukee

- Milwaukee was expanding and starting a master's program.
- Moving to Milwaukee was very fortunate because Dan met and married Mudite, his lovely wife, there.
- During his second year there, Dan received an offer from Wayne State University, and he was moving again.

Wayne State

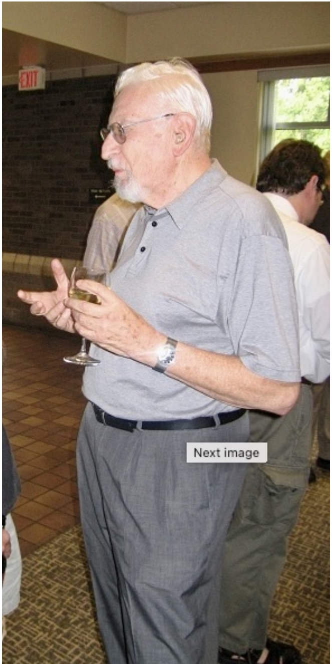
- Wayne State was a stimulating environment with a strong group in analysis.
- Togo Nishiura just finished his doctoral work with Cesari (from Purdue), and started at Wayne State with Dan.
- Dan and Togo formed a fast friendship and a strong mathematical collaboration.

Offer From Syracuse

- While at Wayne State and now with 3 children, Dan received a nice offer from Syracuse University.
- He was attracted by the library at Syracuse, the good environment for his family.
- Dan worked at Syracuse for ~ 30 years and had 11 doctoral students.
- He served as department chair for 7 years, and helped the department immensely.
- He was a real ally to the graduate students!

Syracuse

- Dan thrived at Syracuse.
- The family thrived as well.
- Dan's two daughters, Erica and Susan, went on to become doctors.
- Dan's son Scott studied engineering at Berkeley and got a Ph.D. in Computational Linguistics at Brandeis.

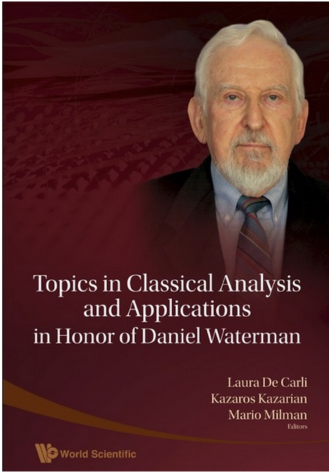


Syracuse

- Dan retired from Syracuse in 1998
- Dan and Mudite moved to Boynton Beach, FL
- Florida Atlantic University offered Dan a position as an affiliated scholar.
- In retirement, Dan consistently did reviews for JMAA.
- Despite problems with his eyesight, Dan continued to publish well into his retirement.

Retiring in Florida

- Dan befriended Laura DiCarli and Mario Milman
- Laura and Mario (and Pam) organized a conference for Dan's 80th birthday.
- This was something that Dan had always wanted, so he was delighted.
- The Proceedings of this conference were published and this was the book cover.



Dan and Mudite's Mathematical Friends

- The Nishiuras
- Yoram Sagher
- The Olevskiis
- The Browns
- Laura DeCarli
- Mario Milman
- The Goffmans
- and so many others...

Thtere were 19 lucky students who got to work with Dan

1. Syed Husain (Purdue) 1959
2. Dan Eustice 1960
3. Donald Solomon (Wayne State)1966
4. Jogindar Ratti 1966
5. George Gasper 1967
6. James McLaughlin 1968
7. Cornelis Onneweer1969
8. Sanford Perlman 1972
9. Arthur Shindhelm (S.U.) 1974
10. David Engles 1974
11. Elaine Cohen 1974
12. Michael Schramm 1982
13. Larry D'Antonio 1986
14. Pedro Isaza 1986
15. David Dezern 1988
16. Nunzio Paul Schembari 1991
17. Hualing Xing 1993
18. Pamela Pierce 1994
19. Franciszek Prus-Wiśniowski 1994

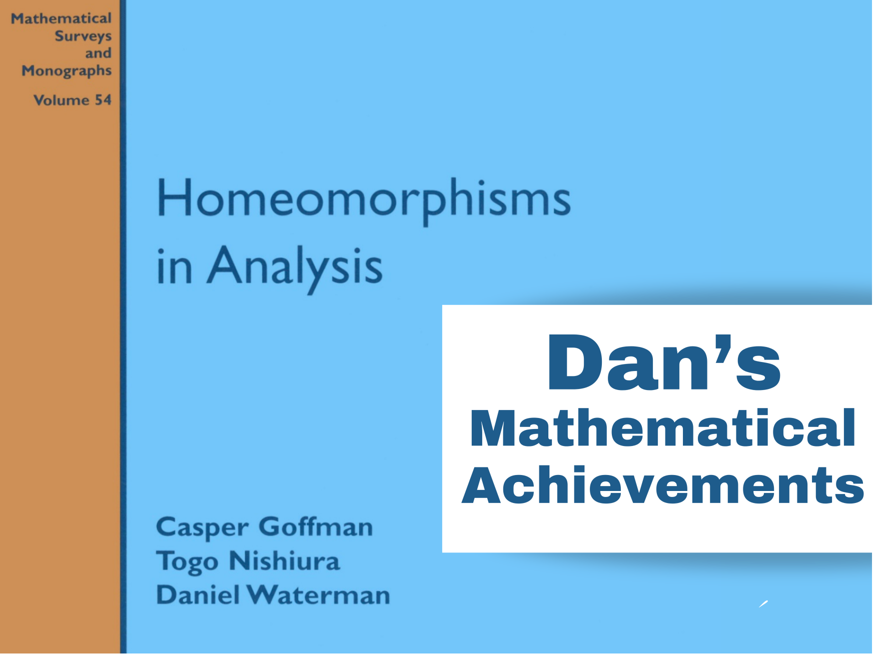
...Dan was viewed by graduate students with an air of mystery, tinged possibly with a little dread. There were rumors of homework questions that didn't have answers in the usual sense, never knowing how seriously to take his suggestions that a few weeks of study might be necessary to understand a single question...
Dan is one of the most caring, giving, and worldly people I have ever met...he taught me that this is a good way to go through life.

-Mike Schramm

Mathematics help: free

Psychiatric help: 5¢

Other Pearls of Wisdom:
PRICELESS



Dan’s Mathematical Achievements

High-Indices Theorems

D. Waterman, *On some high indices theorems*, Trans. Amer. Math. Soc. 69(1950), 468-478

Theorem (Hardy-Littlewood 1924)

If $\sum a_k$ *is Able summable to a number a (that is, if* $\lim_{x \to 1^-} \sum a_k x^{n_k} = a$*) where the sequence* (n_k) *os positive integers has Hadamards gaps (that is,* $\frac{n_{k+1}}{n_k} > q > 1$ *for all k), then* $\sum a_k = a$.

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Setting $x + e^{-s}$, we can equivalently consider the sum of a Dirichlet series $f(s) = \sum a_k e^{-n_k s}$ when $s \to 0+$.

Theorem (Zygmund 1944)

$$\sum |a_k| \leqslant A_q \int_0^\infty |f'(s)|\, ds$$

High-Indices Theorems

Theorem (Waterman 1950)

For $p > 1$

$$\sum |a_k|^p n_k^{p-1} \leqslant A_{qp} \int_0^\infty |f'(s)|^p \, ds$$

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D. Waterman, *On some high-indices theorems II*, J. London Math. Soc. 59(1999), 978-986

Theorem

If $f(s) = \sum a_k e^{-n_k s}$ *is of bounded* Γ*-variation for a* λ*-sequence* $\Gamma = (\gamma_i)$ *and if* (n_k) *is lacunary, then*

$$\sum \frac{|a_k|}{\gamma_k} \leqslant A_q V_\Gamma(f).$$

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A similar result for $f \in \Phi BV$ was obtained in one of the last papers of Dan.

P. Pierce, M. Schramm, D. Waterman, *On some high indices theorems III*, Analysis 28(2008), 1001-1007

High-Indices Theorems

There are two other papers devoted to lacunary series and they establish some Tauberian type theorems.

D. Waterman, *A tangential Tauberian theorem*, Monatshefte f. Math. 65(1961), 101-105

D. Waterman, *A gap Tauberian theorem*, Monatshefte f. Math. 67(1963), 142-144

The Banach-Saks Theorem

Theorem (Banach-Saks)

Every bounded sequence (x_n) in $L^p[0,1]$ or l^p, with $p > 1$, contains a subsequence whose arithmetic means converge strongly.

A Banach space is said to have property $\mathcal{S}$ (property $w\mathcal{S}$) if for every bounded sequence there is a regular summability method T and a subsequence whose T-means converge strongly (weakly).

T. Nishura, D. Waterman, *Reflexivity and summability*, Studia Math. 23(1963) 53-57

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Theorem

For a Banach space X the following statements are equivalent:

(i) *X is reflexive;*

(ii) *X has property $\mathcal{S}$ with essentially positive T;*

(iii) *X has property $w\mathcal{S}$ with essentially positive T.*

The Banach-Saks Theorem

D. Waterman, *Reflexivity and summability II*, Studia Math. 32(1969) 61-63

Theorem

In any Banach space, property $w\mathcal{S}$ with almost regular T^* implies reflexivity, and reflexivity implies $\mathcal{S}$ with positive row-finite column-finite regular T.

Theorem (Zygmund)

If $f \in L^p[0, 2\pi]$, $p > 1$, then for the corresponding Littlewood-Paley functions g and g^*, the following inequalities hold

$$A_p||g||_p \leqslant ||f||_p \leqslant B_p||g||_p$$

(the same for g^*).

Zygmund showed an analogous theorem for the function $s(\theta)$ which is the square root of the area of the image of a special kite-shaped region under a function $f \in H^p$, $p > 1$.

Harmonic Analysis

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Waterman proved similar results for functions analytic in the half-plane:

D. Waterman, *On functions analytic in a half-plane*, Trans. Amer. Math. Soc. 81(1956) 167-194

and for the Marcinkiewicz function μ, extending the results to functions in $L^p(\mathbb{R})$:

Harmonic Analysis

Waterman worked also in more abstract settings:

C.W. Onneweer, D. Waterman, *Uniform convergence of Fourier series on groups I*, Mich. Math. J. 18(1971) 265-273

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C.W. Onneweer, D. Waterman, *Fourier series of functions of harmonic bounded fluctuation on groups*, J. d'Analyse Math. 27(1974) 79-93

D. Dezern, D. Waterman, *On the Legebesgue test for the convergence of Vilenkin-Fourier series*, Mich. Math. J. 39(1992) 425-434

Change of variable

A right system of intervals (at a point):
Let (k_n) be a sequence of positive integers such that $\lim_{n\to\infty} k_n = +\infty$ and $\lim_{n\to\infty} \frac{k_n}{n} = 0$. For each n, let I_{nm}, $m = 1, \ldots, k_n$, be disjoint closed intervals such that for each n, $I_{n,m-i}$ is to the left of I_{nm}. Let there be a real x such that for every $\epsilon > 0$ there is an N such that $I_{nm} \subset (x, x+\epsilon)$ whenever $n > N$. Then the collection $\mathcal{I} = \{I_{nm} : \ n \in \mathbb{N}; m = 1, 2, \ldots, k_n\}$ is called a right system of intervals (at x). A left system is defined similarly.
For every right system $\mathcal{I}$ consider the sequence

$$\alpha_n(\mathcal{I}) \ := \ \sum_{i=1}^{k_n} \frac{(|f(I_{ni}|)}{i}$$

where, for any interval $I = [a, b]$, $f(I) := f(b) - f(a)$.
$\alpha_n(\mathcal{I})$ is defined similarly for left systems.

Change of variable

C. Goffman, D. Waterman, *Functions whose Fourier series converge for every change of variable*, Proc. Amer. Math. Soc. 73(1968) 80-86

Theorem

f is such that $f \circ g$ has an everywhere convergent Fourier series for every homeomorphism g if and only if $\lim_{n\to\infty} \alpha_n(\mathcal{I}) = 0$ for every system $\mathcal{I}$.

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A. Baerstein, D. Waterman, *Functions whose Fourier series converge uniformly for every change of variable*, Indiana Math. J. 22(1972) 569-576

D. Waterman, *Functions whose Fourier series converge uniformly for every change of variable II*, Indiana Math. J. 25(1983) 257-264

Change of variable

Theorem

Let f and g differ only on a set of universal measure zero. Then if one of the functions has a Fourier series converging under any change of variable, so does the other function.

C. Goffman, D. Waterman, *A characterization of the class of functions whose Fourier series converge for every change of variable*, J. London Math. Soc. (2) 10(1975) 69-74

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Theorem

If f is continuous on the circle group T, then there is a homeomorphism g of T such that the conjugate function $\tilde{f} \circ g$ is continuous and of bounded variation.

W. Jurkat, D. Waterman, *Conjugate functions and the Bohr-Pal theorem*, Complex Variables 12(1989) 67-70

Generalized Variation and Fourier Series

D. Waterman, *On convergence of Fourier series of functions of generalized bounded variation*, Studia Math. 44(1972) 107-117

A Λ-sequence: a non-decreasing sequence (λ_n) of positive numbers such that $\sum \frac{1}{\lambda_n}$ diverges.

$$V_\Lambda(f) \; := \; \sup_{\{(I_n\}} \sum_n \frac{|f(I_n)|}{\lambda_n}$$

(λ_n) is bounded iff $\Lambda BV \; = \; BV$.
If $\lambda_n = n$, we get HBV - the family of functions of bounded harmonic variation.

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Theorem (The Dirichlet-Jordan Test)

If $f \in HBV$, then

(i) $S_n[f] \ \to \ f$;

(ii) $S_n[f] \ \rightrightarrows \ f$ *on any interval* $[a, b]$ *consisting of points of continuity of* f.

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D. Waterman, *On Λ-bounded variation*, Studia Math. 57(1976) 33-45

D. Waterman, *Fourier series of functions of bounded Λ-variation*, Proc. Amer. Math. Soc. 74(1979) 119-123

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The new generalization of bounded variation dominated the mathematical interest of Daniel Waterman for the rest of his life.

Theorem

Let(ϕ_n) be a basic sequence of the space S of measurable functions with the topology of convergence in measure. Then after removing any finite number of functions from the sequence, the new sequence remains complete.

C. Goffman, D. Waterman, *Basic sequences in the space of mesurable functions*, Proc. Amer. Math. Soc. 11(1960) 211-213

C. Goffman, D. Waterman, *A remark concerning universal series*, J. Math. Anal. Appl. 40(1972) 735-737

Representations of Functions and Orthogonal Series

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C. Goffman, D. Waterman, *A remark concerning universal series*, J. Math. Anal. Appl. 40(1972) 735-737

K.S. Kazarian, D. Waterman, *Theorems on representations of functions by series*, Mat. Sb. 191(2000) 123-140

Theorem

The completeness of a W-system $\{\psi_n\}$ *is equivalent to the existence of a measure preserving bijection* η *of* $[0, 1]$ *onto itself such that* $\psi_n = w_n \circ \eta$ *a.e. for every n.*

D. Waterman, *W-systems are the Walsh functions*, Bull. Amer. Math. Soc. 75(1969) 139-142

D. Waterman, *On systems of functions resembling the Walsh system*, Mich. Math. J. 29(1982) 83-87

Real Analysis

C. Goffman, D. Waterman, *On upper and lower limits in measure*, Fund. Math. 48(1960) 127-133

The upper limit in measure of a sequence (f_n) of functions is

$$\inf_{(g_n)\in\mathcal{F}} \limsup_n g_n(x)$$

where $\mathcal{F}$ is the set of all sequences (g_n) such that $f_n - g_n \xrightarrow[meas]{} 0$

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C. Goffman, D. Waterman, *Approximately continuous transformations*, Proc. Amer. Math. Soc. 11(1960) 116-121

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M. Schramm, J. Troutman, D. Waterman, *Segmentally alternating series*, Amer. Math. Monthly 121(2014) 717-722